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A MANUAL OF
ELEMENTARY PRACTICAL PHYSICS

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A MANUAL

OF

ELEMENTARY PRACTICAL PHYSICS

BY

JULIUS HORTVET, B. S.

Formerly Teacher of Physics in the East High School, Minneapolis

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THIRD EDITION, REVISED

MINNEAPOLIS
The H. W. WILSON COMPANY
1903

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First edition issued 1900. Second edition revised, enlarged, set up, electrotyped and printed 1902. Third edition printed 1903.

PREFACE TO THE FIRST EDITION

This book has been written in the hope that it will be found helpful in promoting a greater thoroughness in the study of elementary physics. It is an outgrowth of several years experience in teaching experimental physics to large classes in the East High School in Minneapolis. The greater part of the experiments have been performed each year by nearly a hundred pupils, and have been altered until experience has shown that the explanations and directions given were sufficient, and that good results could be obtained. The author, therefore, feels confident that the course described in the following pages will present no unconquerable difficulties to the average high school pupil in the junior year.

It is the plan throughout this book to supply ample necessary explanations as to the theory and purpose of each experiment, so that the significance of each experiment and its results may be more fully perceived. It is not expected that the pupil shall go through the necessarily slow process of acquiring *all* his knowledge by his own investigations; *some* information must be given him, in fact, in many cases it is almost inevitable that in giving such directions as will enable a pupil to discover a fact, the fact shall be stated beforehand. While in theory much is conceded in favor of inductive methods in science teaching, it is found in practice that the *purely* inductive method fails at points where it is expected to do the greatest amount of good. Life is not long enough to admit of a *rediscovery* of the fundamental laws of physics. Besides, an important fact that seems to be generally overlooked is that some of the great laws were not discovered through experiment at all, but, on the contrary, were obtained by pure reasoning, and afterwards *verified* by experiment.

The comparatively small value of mere book knowledge in any of the experimental sciences is conceded by all who realize the purpose of scientific study. The knowledge of facts is not the be-all and end-all of study in any subject. To reap the full benefit of the mental discipline that the study of physical science offers, reading must go hand in hand with individual experiment. The pupil must learn by his own experience and observation what experimental evidence means, in order to appreciate rightly the evidences and the processes of reasoning by which the fundamental laws of physical science are established.

The purpose of this course is to teach pupils to measure accurately, to manipulate carefully, to work methodically, to see fully, to reason intelligently, and to express their observations and results clearly. Each pupil is expected to think for himself and to treat each experiment as a problem which he himself has to solve, while the chain of reasoning to be followed and the conclusions to be drawn are indicated by questions which the pupil has to answer. If the pupil cannot answer certain questions, the teacher can induce him to answer them by other questions judiciously put. In brief, after a pupil is fairly prepared to undertake a given experiment, he is compelled thereafter at every step to *think* in order that he may realize the richest returns.

The keeping of a note-book is regarded as an important part of a course in practical physics. The work of preparing a proper record of each experiment, both in temporary and permanent form, is in itself a valuable mental discipline. In fact, a course in experimental physics that does not encourage the keeping of a good note-book is far from fulfilling its purpose. In this course the pupil is expected to keep systematic notes, and receives careful instructions as to how all notes should be recorded.

The course includes about forty experiments, and can be completed in a school year of nine months, allowing an average of two hours a week for the laboratory work. In the selection of the experiments and methods of procedure, attention has been given mainly to those fundamental principles which are considered well calculated to give the pupil a good preparation for the further study of physics. While in none of the experiments is there anything too advanced for the average pupil of the age that has been mentioned, it can be said that no effort has been made to make anything *easy* to the extent of depriving the pupil of knowledge and discipline that will be all the more valuable for the extra effort that may be demanded.

The matter of apparatus cannot longer be considered a serious obstacle to the beginning of good laboratory work. Apparatus suitable for the experiments in this book can now be obtained at reasonable prices. A list of the apparatus required for the laboratory work is given in the Appendix, under Apparatus and Materials. The author will be pleased to give any information on this subject to teachers who will write to him.

The author desires to express his indebtedness to other teachers of physics who have given him many valuable suggestions. Especial thanks are due to Mr. A. Zeleny of the University of Minnesota for reading portions of the manuscript and making helpful criticisms.

Nearly the entire course of experiments contained in this book has been given in recent years to classes in elementary physics in the University of Minnesota Summer School.

J. H.

MINNEAPOLIS, MINN.
March, 1902

PREFACE TO THE THIRD EDITION

In preparing a new edition of this book an effort has been made to do three things: first, to bring the subject matter itself up to date; second, to make, wherever possible, the discussion of theory and the explanation clearer than before; and, third, to bring the book as a whole into closer correlation with several of the best text-books on elementary physics. It is believed that the physical meaning of all relations is now shown so that all pupils who have had the usual first two years of high school mathematical training will be able to understand and apply them. Several sections, notably in Light and Electricity, have been entirely rewritten and some new experiments and exercises have been introduced throughout the book. There are now three appendixes, the first being a description and detailed specifications for making a student's physical laboratory table, the second giving advice as to the selection of apparatus, together with a full list of the material and apparatus needed for the book, and the third containing twenty carefully prepared tables of physical constants.

Minneapolis, Minn.
September, 1903.

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GENERAL INSTRUCTIONS

Preparation.— Before beginning any particular experiment it is essential in the first place to understand clearly the object of the experiment and the theory involved. Read carefully through the instructions and explanations in order to realize fully what measurements are to be made and how the results of these measurements are to be treated. If certain principles are unfamiliar, they should be looked up in some reference book before the experiment is begun.

Apparatus.— Become familiar with the construction and use of each piece of apparatus to be employed in the experiment. Eagerness to "use" a piece of apparatus before it is properly fitted up and tested often leads to results that are bad both for the pupil and the apparatus. When there is uncertainty as to the practical handling of the apparatus or as to the nature of the results to be obtained, it is sometimes necessary to make a set of preliminary observations as a part of the preparation for the experiment.

Mistakes.— Mistakes ought never to happen and must be carefully guarded against. An illustration of the way mistakes occur may be given. There is a tendency, while paying special attention to the smaller details of an observation, to make a mistake in noting the numbers which express the larger part of the result. Thus, in measuring a length, attention may be concentrated on the millimetres while a mistake is made in noting the number of centimetres. When it is known that a mistake has been made in an observation, the result should be rejected. A result should not be rejected simply because it differs from other results obtained in the same set of observations.

Errors. — All physical measurements are subject to errors. Absolute accuracy can in no case be expected; yet an effort should always be made to reduce errors to a minimum. To eliminate errors completely is quite impossible. All errors may be grouped as follows:—

(1) *Systematic Errors,* which are quite beyond control and are scarcely to be diminished however often the observations are repeated. They are due to such causes as currents of air and changes in temperature.

(2) *Constant Errors,* the causes of which may be in the indications of the instrument or in a faulty method of taking observations. These errors always preponderate in a definite direction, making the result always either too large or too small. As they are due to known causes, they may be practically eliminated, usually by making suitable corrections or by employing radically different methods.

(3) *Accidental Errors,* due to unknown causes. An observer making two measurements of the same quantity, with the same instrument, and apparently under the same conditions, may in most cases obtain results which disagree to an appreciable extent. Errors of this kind give results that are as apt to be above as below the exact value.

Observations. — Regarding the number of observations which should be made in each case, no general rule can be given. The effect upon the result of accidental errors may to a great extent be eliminated by taking the mean of several observations. The arithmetical mean of the results obtained by repeated experiments is probably more correct than any one result by itself. Every observation made should be wholly uninfluenced by any previous observation or by an unconscious tendency to make all results agree. Let each observation be taken as carefully as though the final result depended upon it alone.

"In higher experimental work, whether research or practical, the investigator is determining a quantity which is to him unknown. It is desirable to reproduce this condition, as far as possible, in an elementary laboratory course. Therefore the student should not ask himself whether his results are correct and in agreement with known values, but whether his method is correct and his observations well taken." (W. C. Sabine.)

"It is to be remembered that the object of scientific observations is not to confirm preconceived theories, or to obtain a series of results which shall arouse admiration on account of their uniformity, but to discover the truth in regard to the phenomena investigated, no matter what the truth may be. It is of the greatest importance, therefore, that the observer should be entirely unprejudiced, either by a knowledge of the results of other experimenters, or by any preconceived notion as to what the results should be. It is not meant by this that the observer must be ignorant of the probable results; but that his observations should be taken with as much care as though he were ignorant; and that great precautions must be taken to avoid the almost unconscious tendency, to which all observers are more or less subject, of making the observations correspond with what is thought to be the truth."
(E. L. Nichols.)

Relative Importance of Observations.—While it may be desired that the result of an experiment be highly accurate, it does not follow that all observations should be taken with the greatest possible care. If errors in one set of observations will produce a large error in the result, then these observations must be taken with special pains. If, on the other hand, errors in another set of observations will produce only a slight effect on the result, it will be needless to devote much time and effort to the observations. Those observations which are liable to contain the greatest percentage of error will produce the greatest error in the final result, and must therefore be taken with the greatest care. To find which set of observations is liable to the greatest percentage of error, reason as in the following examples:—

(1) Suppose it is required to measure the area of a rectangle whose sides *a* and *b* are respectively about 2 and 90 centimetres in length. If the smallest divisions on the scale are 0.1 centimetre in length and if the tenths of these divisions be obtained by estimation, all readings can be taken to within 0.01 centimetre. An error of 0.01 centimetre in the measurement of side *a* is an error of about 1 part in 200 and a similar error in the measurement of side *b* is one part in 9000. It is thus seen that the same error in the two measurements will introduce a greater percentage of error in the measurement of the shorter side. This side, therefore, must be measured with the greater care.

(2) Suppose it is required to measure a mass and a change of temperature, the mass estimated to be about 400 grammes and the change of temperature about 4 degrees. Suppose the balance used is sensitive to 0.01 gramme, and that estimations on the thermometer can be made to 0.1 degree. The weighings can then be made to within 1 part in 40,000 and the change of temperature can be read to within 1 part in 40. It is thus seen that the greater percentage of error occurs in the measurement of the change of temperature. An error of 10 grammes in the

measurement of the mass will be equivalent to an error of 0.1 degree in the observation of the temperature. Hence, so far as the effect upon the result is concerned, it is wholly unnecessary in this case to determine the mass even to tenths of a gramme, although with a little care it could be done.

Note-Books.—Results should be written down, at the time they are obtained, in a convenient note-book kept for the purpose. The original observations should never be trusted to loose sheets of paper, backs of envelopes, etc., even though it be the intention to copy them in a note-book afterwards. It is of the greatest importance on all grounds that a neat, orderly, and accurate record of all work done should be preserved. For this purpose two note-books should be kept, a smaller one for the temporary record of the observations and calculations, and a larger one for the complete, permanent record of all the experiments.

Enter in the temporary note-book:—

(1) Every actual observation which may have a bearing on the final result, including date of the experiment, and where necessary, the number of each piece of apparatus used.

(2) All necessary calculations. Arithmetical processes should be written out completely and in good form, in order that the work may readily be reviewed and mistakes easily traced.

(3) A temporary, though orderly, tabulation of the results. Observations should not be entered at random over the page, but in such a way that at any future time the meaning of each entry will be perfectly clear. The habit of making the temporary record of an experiment neat, compact, and legible should be cultivated. The notes should be frequently seen by the instructor, and

should not be written up in permanent form before they have been fully discussed and finally passed as satisfactory.

The permanent record of each experiment should be in ink.

Enter in the permanent note-book:—

(1) The number of the experiment.

(2) The date on which the observations were taken.

(3) The object of the experiment.

(4) A working description and neatly drawn figure of the apparatus. Give also the number of each piece when required.

(5) A statement of the sources of error.

(6) A brief description of the method of procedure.

(7) Each observation made, and all results in a properly tabulated form.

(8) Conclusions to be derived from the results, briefly stated or indicated.

(9) Numerical computations, showing only necessary steps and formulæ.

(10) Answers to questions and problems suggested by the experiment and illustrating the principles involved. The solutions of problems should be given in full, and the answers to questions given in such a manner that there will be no trouble recalling the nature of each question.

Make all descriptive notes so clear that any person of ordinary intelligence can readily understand them. Use the shortest and simplest words possible. Make the language mean exactly what it is intended to mean. Bear in mind always that a plain, straightforward style of discussing an experiment is one of the most important aids to clear thinking, and that precision of statement and scientific truthfulness of expression are habits well worth cultivating. Some faults to be avoided are, (1) the use of incorrect English, (2) the use of the imperative mood, (3) false reasoning and absurd statements, (4) mathematical errors, (5) the use of common fractions instead of decimals, (6) frequent erasures.

Significant Figures.— A distinction should always be made between real and apparent accuracy, and it is well to remember that the degree of accuracy of any calculated result is limited by the instruments and methods used. In writing down the results of observations and calculations, care should be taken to show precisely the extent of the accuracy attained. The two numbers 3.4 and 3.400 do not mean the same thing when put down as the result of a measurement. The former shows the result obtained to two significant figures, and no attempt made to determine the third, while the latter shows the result obtained to four figures with the two last figures found to be zero. If in measuring a length by means of an instrument easily capable of giving the result to tenths of a millimetre, the result should happen to be exactly 5 centimetres, it should be written as 5.00 centimetres, or 50.0 millimetres.

There is nothing to be gained by introducing into a result an imposing array of decimal figures extending beyond the limit of probable accuracy. To carry a calculation out to six or seven places of decimals, when probably the result can not be accurate to the second place, is not only time wasted but it is also misleading. The quotient obtained by dividing 24.65 by 3 might be written 8.21666 indefinitely, implying that the mere process of continued division by 3 tends to extend the limit of certainty of the result. As a rule, the number of places of figures to be included in a result will be one more than the last figure whose value is reasonably certain. All figures retained as certain are said to be significant; the last figure is doubtful and is underlined. As examples, take $13\underline{8}$ and $.00013\underline{8}$, both given to two significant figures. To illustrate further, take the following data from a note-book:—

(I)	(II)
4.34 cm.	4.36 cm.
4.32 "	4.35 "
4.36 "	4.36 "
4.33 "	4.34 "
Mean = $4.3\underline{3}$ cm.	Mean = $4.35\underline{2}$ cm.

A figure may be considered significant if its value is reasonably certain within one unit. In case I, the figures obtained in the third column vary from 2 to 6, and the third figure in the mean appears to be in error by about three units. The figure 3 is underlined. In case II, the figures in the last column vary from 4 to 6, and the third figure in the mean, being certain within one unit, is retained as significant. It is clear that the figure 2, obtained by division, is doubtful. When the results given by a set of measurements vary as in case I, the mean may be made more certain by taking a large number of measurements, say 15 or 20.

The sum of a significant figure and a figure that is not significant must be doubtful. A like rule applies to their difference. As examples, take:—

(Sum)	(Difference)
$8.2\underline{4}$	8.24
$.32\underline{4}$	$.3\underline{2}$
$8.5\underline{6}$	$7.9\underline{2}$

When several factors are involved in a calculation, the final result can never be more accurate than the least accurate factor. The percentage error introduced into the result by any factor is equal to the percentage error in that factor. Retain in the result as many significant figures as there are in the least accurate factor, when the first figure of the result is equal to or greater than the first figure of the least accurate factor. When the first figure of the result is smaller than the first figure of the least accurate factor, one more figure is retained and marked doubtful. As examples, take the following:—

$$\begin{aligned}
2.3\underline{2} &\times 95.6\underline{3} = 22\underline{2}. \\
16.34\underline{2} &\times 0.8\underline{3} = 13.\underline{5} \\
42.5\underline{6} &\div 5.1\underline{2} = 8.3\underline{1} \\
42.5\underline{6} &\div 0.9\underline{3} = 45.\underline{8} \\
64.\underline{2} &\div 432\underline{1} = 0.000148\underline{5}
\end{aligned}$$
(three significant figures)

Estimation of Percentage of Error of a Result.—In many cases the numerical values obtained as results of experiments may be tested by comparison with data furnished in tables of reference. Simply to measure the accuracy of a result by the *difference* between the correct result and that which is obtained is not sufficient. An adequate expression of the degree of accuracy can be obtained only by finding what fraction (expressed as a percentage) this difference is of the exact or approximate standard value. The fraction is changed to the percentage form by multiplying its decimal expression by 100, thus:—

$\frac{1}{4} = .25 = 25$ per cent.
$\frac{1}{25} = .04 = 4$ " "
$\frac{3.4}{80} = .0425 = 4.25$ " "

To illustrate further, take as an example the result obtained in a measurement to determine the number of centimetres in one inch The mean of all the observations taken is $2.52\underline{4}$ cm. The difference between this result and the correct value is $2.54 - 2.52\underline{4} = 0.01\underline{6}$ cm. The error expressed in per cent. equals

$$\frac{0.016}{2.54} \times 100 = \frac{1.6}{2.54} = .6, \text{ nearly.}$$

If the measurement of a quantity has been repeated many times and the same care given to each measurement, the arithmetical mean of the results may be taken as the standard of comparison. By comparing this mean value with each individual measurement, the degree of accuracy of the observations may be estimated and the errors expressed in per cent. From a consideration of these percentage errors and the number of times the measurements were repeated, an idea may be formed as to the probable accuracy of the mean value.

There is no uniform rule by which the degree of accuracy which ought to be attained in the experiments described in this book can be governed. This point can be decided in some cases only by an exercise of judgment, taking into consideration the apparatus used and the conditions under which the experiment is performed. In general, the results obtained should show an error of not more than one per cent. In all experiments it is expected that pupils will seek to obtain fair results, and to that end, they may find it necessary sometimes to repeat the observations more often than the directions seem to require.

Treatment of Related Quantities.—An inspection of the numerical values obtained as results of an experiment will usually reveal a definite relation, or law, connecting the quantities measured. The relation between two variable quantities is generally expressed as a proportion. The values of one quantity may be proportional directly or inversely to the squares of the others, or to the square roots. Take as illustrations the following:—

(*a*) The volume of a gas is proportional to its absolute temperature.

(*b*) The intensity of illumination at a point due to a given source of light is inversely proportional to the square of the distance of the point from the source.

(*c*) The time of vibration of a pendulum varies directly as the square root of the length.

One quantity is said to be proportional to another if the two quantities are so related that the result of dividing one by the corresponding value of the other is always the same. Thus, let m, m' and n, n' be the values of two related quantities under certain conditions given in an experiment. The law which connects the two quantities may be expressed as follows:

$$\frac{m}{n} = \frac{m'}{n'},$$

and

$$m : m' = n : n'.$$

The result of dividing one quantity by another is called the ratio of the first to the second. When the ratio is constant the relation between the two quantities is shown to be direct.

One quantity is said to be inversely proportional to another when the two are so related that an increase in one is accompanied by a decrease in the other, in such a way that the result of multiplying the corresponding values of the two quantities together is always the same. Thus, let m, m' and n, n' be the values of two related quantities under certain conditions, and let k be a constant factor.

Then

$$m = k\,\frac{1}{n} \text{ and } m' = k\,\frac{1}{n'},$$

and

$$m\,n = k \text{ and } m'\,n' = k.$$

Whence, the relation between the two quantities may be expressed as follows:

$$m\,n = m'\,n',$$

and

$$m : m' = n' : n.$$

Then, to find whether the one quantity varies directly as the other, compute their quotients; to find whether the one varies inversely as the other, compute their products. If neither quotients nor products are equal, compute the squares or the square roots of the values of one of the quantities, and try the quotients or the products as before.

Position of a Point in a Plane.—The geographer describes the position of a town by referring to the position of some known point on the earth, called the *point of reference.* The point of reference for the surface of the earth is the intersection of the meridian of Greenwich with the equator. The location of a town is definitely described when two things are known about it; viz., its *latitude* and its *longitude.* Thus, the city of Havana is 23° 0′ north latitude and 82° 20′ west longitude.

By a similar method, the position of a point P (Fig. 1) with reference to another point O in the same plane is described when two straight lines are drawn forming right angles at the point O, and the perpendicular distance of the point P from each of these two lines is given.

The point O is called the *origin;* the horizontal line of reference OX, *the axis of* X; and the vertical line of reference OY, *the axis of* Y. The longitude x of the point P is called its *abscissa;* while the latitude y is known as its *ordinate.* Abscissae are positive when drawn to the right from the axis of Y, negative when drawn to the left. Ordinates are positive when drawn upward from the axis of X; negative when drawn downward.

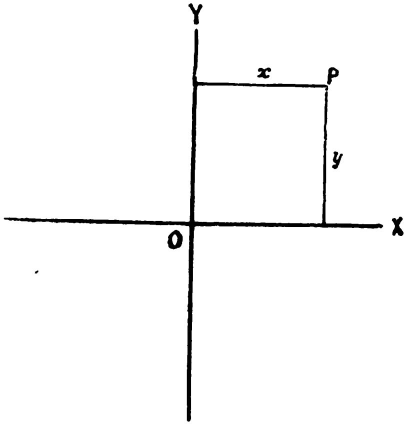

Fig. 1.

Curve Plotting.—The significance of the results of an experiment can often be shown very clearly if expressed graphically by means of a curve. In constructing the curve it will be found best to use a sheet of cross-section paper. This paper is divided into small squares by two sets of parallel lines ruled at regular intervals from each other—say, for example, one-tenth of an inch, or one millimetre. It is a matter of convenience to have every fifth or tenth line heavier than the intervening ones, or of a different color. The tenths of the smallest spaces can be estimated.

As an example of the use of the graphic method, take the following values showing the relation between the boiling temperature of water and the pressure in centimetres of mercury:

Pressure.....	70.7	72.3	73.3	74.9	76.0	77.6	78.7
Temperature.	98.0	98.6	99.0	99.6	100.0	100.6	101.0

On the cross-section paper a horizontal line is selected as the axis of X, and a vertical line as the axis of Y. It is usual to choose the heavy line near the bottom of the sheet for the axis of X, and the one near the left-hand edge for the axis of Y. An important point to be attended to is the fixing of the scales according to which the axes are to be divided. The best scale will depend in each case somewhat upon the range of the values to be plotted. As a rule, it is desirable to adopt for each axis a unit of such magnitude that the curve will as nearly as possible occupy the whole of the sheet. Also, it will be found convenient in most cases to choose the smallest division on the paper to represent the last significant figure in each measurement. In the example given, a suitable scale to adopt for the horizontal axis would be fifty divisions to correspond to a degree of temperature, and for the vertical axis, ten divisions to correspond to a difference of one centimetre in pressure. In marking the scales the actual numbers to be plotted need not be shown. On the horizontal axis, OX, mark the point O as 98, the fifth heavy line 99, the tenth 100, etc. On the vertical axis, OY, beginning at O, mark 70, 72, 74, etc., omitting to mark every alternate second line. Along each axis place small dots at the locations of the values to be plotted. Now mark with very small crosses the points obtained by the intersections of vertical lines from the points marked on the axis of abscissae with horizontal lines from the points marked on the axis of ordinates. These represent the pressures and temperatures respectively and are called the *co-ordinates* of the points located on the sheet. Since the

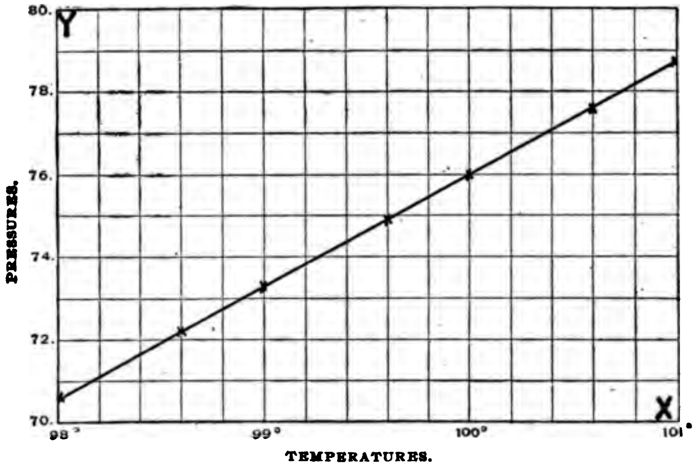

Fig. 2.

paper is ruled, it will never be necessary to draw the perpendiculars to the axes; the co-ordinates can easily be traced by referring to the ruled lines. A continuous line drawn evenly through the crosses will represent to the eye the relation between the quantities. If it is impossible to pass the line evenly through all the crosses, the line can be drawn in such a way that the crosses lie as near to it as possible, that is, so that there shall be as many on one side of the line as there are on the other.

If the line drawn through the crosses be straight, it can be shown that the quantities in question are directly proportional. Thus, referring to Fig. 3, OAC and OBD are similar triangles and OC:OD::AC:BD. Or EA:FB::AC:BD. If, then, a curve be plotted in such a manner that AC and BD represent pressures and EA and FB represent temperatures, it can be seen that the temperature is proportional to the pressure. Where the line is a curve, the law expressing the relation between the

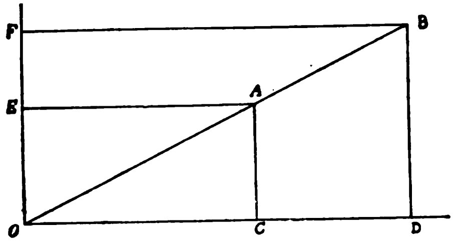

Fig. 3.

quantities is more complex. In the case of Boyle's Law, the curve showing the relation between the pressure and the volume of a gas is a rectangular hyperbola. In a curve of this kind the rectangle under the co-ordinates of any point on it is a constant quantity. This is the same as saying that PV = a constant. If the values of P and $\frac{1}{V}$ be plotted, the line connecting them will be found to be straight. The graphic method not only points out where mistakes have been made by the observer, but also enables one to get approximate values at any intermediate points which have not been determined directly by the experiment. If nearly all the crosses be well on the line, while one or two are at a considerable distance from it, it is evident that errors were made in the observations on which the positions of these crosses depend.

THE FUNDAMENTAL UNITS

Modern physics is essentially quantitative in character. So far as man's knowledge of the phenomena of nature has become accurate, it has been obtained by means of precise measurement. A physical *measurement*—a measurement, that is to say, of a physical quantity—consists in comparing the quantity to be measured with a unit quantity of the same kind. Comparison is here understood to mean the process of finding the number of times that the unit is contained in the quantity measured. Nearly all physical quantities are measured in units which in practice are referred to the three fundamental units, viz., the unit of *length,* the unit of *mass,* and the unit of *time.* All other units needed in physical measurements are defined in terms of these units, and are, therefore, called derived units.

The system of units now universally employed in physical measurements is based upon the *centimetre* as the unit of length, the *gramme* as the unit of mass, and the *second* as the unit of time. This system was proposed in 1863 by a Committee of the British Association, and is now known as the absolute C. G. S. (centimetre-gramme-second) system.

In practical physics another absolute system is sometimes used, based upon the foot, the pound, and the second as the fundamental units of length, mass, and time. This system is called the foot-pound-second system.

The Unit of Length. — The centimetre is the one-hundredth part of the length of the standard *metre.* The standard metre is the distance between two marks on a certain platino-iridium bar preserved in the International Metric Bureau at Sevres, near Paris. It was the intention in constructing this standard to have its length precisely equal to one ten-millionth of the distance between the earth's equator and the north pole measured along a meridian. Subsequent surveys of the earth showed that the length of this bar does not correspond exactly with its intended length. Recent measurements give for the length of the earth's quadrant 10,002,015 metres. Yet, for practical reasons, the bar is still regarded as the standard metre, and from it all other metres have been directly or indirectly copied.

The Unit of Mass.—The gramme is the one-thousandth part of a standard mass of platinum, called the *kilogramme,* preserved in Sevres. This standard was intended to be equal in mass to one cubic decimetre of pure water at 4°C. More exact determinations have proved that the true mass of a cubic decimetre of water at the above temperature is 1.000013 kilogramme, so that, for most practical purposes, the standard may be taken to agree with the value which it was originally intended to have.

The Unit of Time.—The second is $\frac{1}{86400}$ of a *mean solar day.* The solar day is the interval between two successive transits of the sun's centre across the meridian of any given place. But this interval varies in length from day to day on account of the varying rate of motion of the earth in its orbit. The mean length of the intervals for one year is, therefore, taken in order to establish the standard of time called the mean solar day.

MEASUREMENT OF LENGTH

THE LINEAR SCALE

It can be shown that every physical measurement involves in one way or another a measurement of length. Hence it is of the greatest importance to attain a high degree of accuracy in linear measurements. Measurements are always made by the use, directly or indirectly, of a divided scale, which is usually a copy of a whole or a part of a standard. Fig. 4 is a copy of a steel scale as used in the laboratory. It is about a foot long and divided into fractions of a metre and of the foot. On one side of this scale the long numbered lines show centimetres, and the smallest divisions are millimetres.

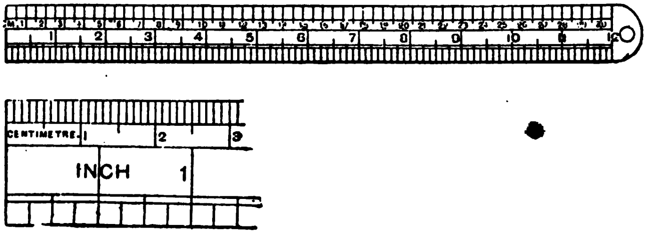

Fig. 4.

Abbreviations.—Those commonly employed are:

Metre,	m.
Decimetre,	dm.
Centimetre,	cm.
Millimetre,	mm.

Use of the Scale.— In measuring a given length by means of a scale, the scale-edge should be placed as [illegible] establish the [illegible] day.

MEASUREMENT [illegible]

[illegible]

near as possible to the ends whose positions are to be determined. If in any case the scale cannot be so closely applied, as when it is necessary to sight across the thickness of a rule, it is important that the line of sight be made perpendicular to the scale. Thus, as shown in Fig. 5, a slight displacement of the eye to either side of the perpendicular AB drawn to the point at which a reading is to be taken will make the reading either too great or too small, according as the eye is in position D or position C. The error introduced is said to be due to *parallax*. In this case, parallax means the angle between a given line of sight and the line perpendicular to the scale, or simply the shifting of the point of view. To avoid parallax, some simple means may often be provided to aid in finding the correct line of sight. One device consists of a scale etched in front of a mirror. If the eye is so placed that an end whose position is required covers its image in the mirror, the correct position of the eye has been found.

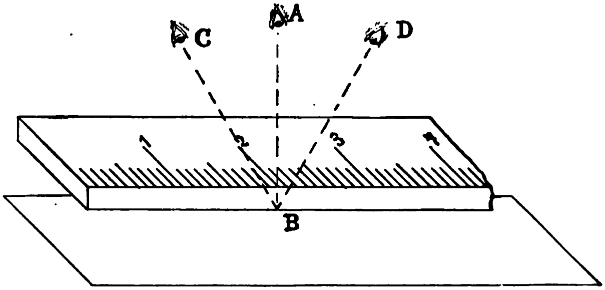

Fig. 5.

In general, it is not best to start at the end of the scale in making a measurement. The ends of a scale are seldom cut square, and an end division is generally inexact. Apply the scale to the length to be measured so that all readings can be taken within the ends of the scale. Let one end of the length coincide with one of the centimetre divisions, and take the reading of the scale opposite the other end. From this reading and the reading at the other end the length can be found.

The Estimation of Tenths.—It will rarely happen that after the scale has been placed with one of the centimetre divisions exactly on one end of the length to be measured, the other end will coincide precisely with a scale division. On this account it becomes necessary to complete a reading by the estimation of tenths. It is not difficult to imagine a millimetre space divided into ten equal parts and to express the position of a point as so many centimetres, so many millimetres, and so many tenths of a millimetre. In Fig. 6, for example, the line *ab* would have for its reading 12.42, because the distance between 12.4 and 12.5 has been divided into 10 equal parts by estimation, and 2 of these parts are estimated to the left of *ab*. In the same way the position of *cd* is noted as 42.85.

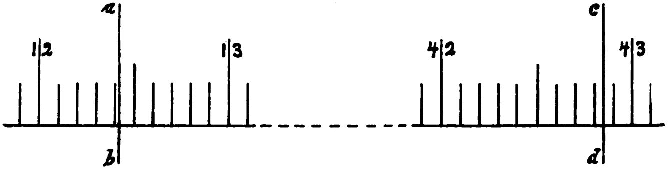

Fig. 6.

As a rule, in every case when a number has been written down as the result of an observation, show its denomination, thus: cm., mm., or in., as the case may be. In taking readings on the English scale, express the fractions of the inch in the form of decimals, thus: $\frac{1}{8} =$

0.125, $\frac{1}{12} = 0.0833$, etc. The tenths of the smallest division of the inch should be estimated.

To determine the length of a cylinder in centimetres, the same length in inches, the number of centimetres in one inch.

Apparatus: (1) A metric scale. (2) An English scale. (3) A metal cylinder. (4) A magnifying lens.

Errors: (1) The scale and the axis of the cylinder may not be parallel. (2) In determining the exact position of the end of the cylinder on the scale there may be an error due to parallax. (3) There may be defects in the scale itself.

Procedure: Place one end of the cylinder so as to coincide with a division of the scale, and in such a position that the other end will extend into the fractional divisions. (See Fig. 7.) Read the position of each end of the cylinder, estimating the tenths of the smallest division. The accuracy of the readings may be increased by the aid of a lens. Make in all five sets of observations, each time placing the cylinder over different sets of scale divisions. Find the average of the results, so as to obtain the most probable length of the cylinder. In like manner and with the same care, measure the length of the cylinder with the English scale, expressing the fractions

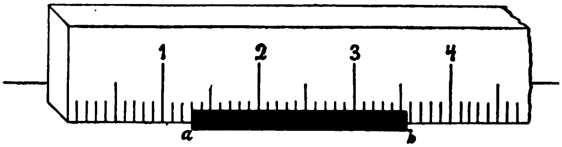

Fig. 7.

of an inch as decimals and estimating the tenths of the smallest division. Tabulate the results as follows:

Cylinder No. **Scale No.**

Metric.			**English.**		
End *a*.	**End *b*.**	**Length.**	**End *a*.**	**End *b*.**	**Length.**
cm.	**cm.**	**cm.**	**in.**	**in.**	**in.**

Mean length, cm. **Mean length, in.**
Number of centimetres in one inch,

Express this last result also in millimetres, decimetres, and metres.

Exercises: (1) Using the results obtained, compute (*a*) the number of inches in a metre, (*b*) the number of metres in a mile.

(2) What would be the probable effect on the result if the length measured were made much greater?

(3) What is the reason for using a different set of scale divisions in each trial?

(4) Make a plain drawing, full size, of a short section of each scale, showing as accurately as possible the relative sizes of the divisions.

(5) Which is the greater, 20 centimetres or 8 inches? Express the difference in millimetres and tenths of a millimetre.

THE VERNIER

In measuring a length by means of a scale, it often happens that while one end may be made to coincide with a division line, the other end will fall somewhere between two divisions. With ordinary scales, under favorable conditions, it has been shown that it is possible to obtain the tenths of the smallest division by estimation. Readings of much greater accuracy can be obtained by means of a contrivance called a *vernier*. This consists of a series of equal divisions on one or both sides of a sliding index. The index line is the zero of the vernier, while the other lines are used to aid in determining the exact position of the index on the scale. The divisions are usually numbered from the zero line in increasing order in the direction of the ascending numbers of the scale. The vernier divisions are made to differ in length from those of the scale, in every case in such a manner that n vernier divisions are equal to $n + 1$ or $n - 1$ scale divisions. In every case each vernier division is $\frac{1}{n}$ of a scale division longer or shorter than each scale division.

To illustrate the use of a vernier, take the simple case in which 10 vernier divisions equal 9 scale divisions. This makes each vernier division shorter than a scale division by one-tenth of a scale division. Suppose, as in Fig. 8, the zero line of the vernier is made to coincide with the end of a length to be measured and is found to lie between two scale divisions. Read the last division of the scale below the zero of the vernier. Find the line of the vernier which coincides with a division line on the scale. The number of this gives the number of tenths of a scale division desired. To illustrate further, suppose the zero of the vernier is between 10.2 and 10.3 on the scale, also that the line 5 on the vernier coincides with a line 10.7 on the scale. Then, since each vernier division is one-tenth shorter than the smallest division of the scale, the distance between

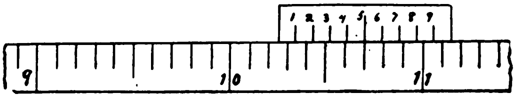

Fig. 8.

4 on the vernier and 10.6 on the scale equals 0.1 scale division.
3 " " " " 10.5 " " " " 0.2 " "
2 " " " " 10.4 " " " " 0.3 " "
1 " " " " 10.3 " " " " 0.4 " "
0 " " " " 10.2 " " " " 0.5 " "

Since 0.5 of the smallest scale division is 0.05 of the larger division, the reading is 10.25.

If no vernier line coincides with a line on the scale, an estimation by the eye must be made as to where the coincidence would occur if the scale were further subdivided. Generally, however, it is sufficient to take the mean of the readings of the two lines which are nearest to coincidence.

The Vernier Caliper. — There are many kinds of measuring instruments in each of which the vernier is an essential part. One of these, known as the vernier caliper (Fig. 9), consists of a graduated metal scale having a fixed jaw at one end and at a right angle to it another parallel jaw fixed to a sliding piece on which a vernier is engraved. When the jaws are together, the zero of the vernier should coincide with the zero of the scale. Hence, when the jaws are open, their distance apart will be given by the position of the zero of the vernier on the scale. A

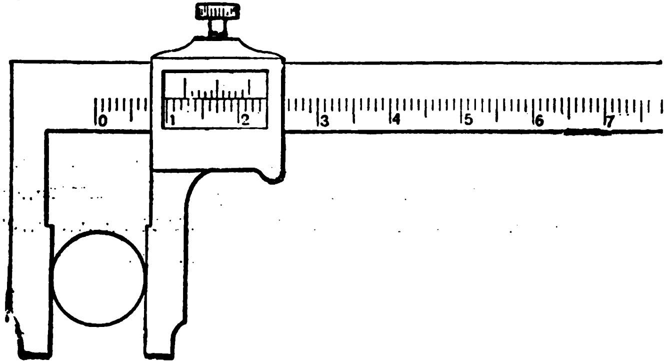

Fig. 9.

set screw, attached to the movable jaw, enables it to be set in any position desired.

To measure the linear dimensions of a metal cylinder and to find its volume.

Apparatus: (1) A vernier caliper. (2) The metal cylinder used in the preceding experiment.

Errors: (1) The inner faces of the jaws may not be parallel. (2) The length to be measured may not be parallel to the scale. (3) Excessive pressure may spring the jaws. (4) A dimension may vary at different parts of the cylinder.

Procedure: Examine the scale on the limb to determine the value of a unit division. See that the inner faces of the jaws are clean. Before making a measurement, close the jaws to determine the zero reading. Holding the caliper between the eye and the light, and looking between the jaws, bring them together at all points as near as possible. Clamp the sliding jaw, turning the screw lightly, and note the line of the vernier which coincides with a line on the scale, taking great care to avoid parallax. The number of this line gives the zero reading to the tenths of the smallest scale division. In any case this result virtually reads from a division line — 1 next below the zero of the scale. For example, if this reading is 0.8, the zero of the vernier must be 0.2 below the scale zero, hence the zero reading will be written — 0.2. When the zero of the vernier is above the zero of the scale, the zero reading is written with the positive sign. The difference between a zero reading and another reading taken elsewhere on the scale in measuring a dimension will give the result required. Hold the jaws again to the light and note at what part they are in closest touch. Use this part in the measurements to be made. Having obtained the zero reading, open the jaws and insert the cylinder so as to measure its length. Hold the cylinder so that its axis is parallel to the limb. Bring the jaws to bear against the ends of the cylinder. Clamp the vernier and take the reading. Repeat the measurement four times, each time with the ends of the cylinder in a different position against the jaws. Take a new zero reading before each trial. Subtract the mean of the zero readings from the mean of the other readings on the scale to find the average length. In the same way measure the diameter of the cylinder. Repeat the zero reading and measure in a different place and direction each time throughout the whole length. Using the mean value of each dimension, compute the volume of the cylinder by the formula $\pi r^2 l$, in which r is the radius and l the length. Tabulate as below:

Cylinder No. **Caliper No.**

Length.		Diameter.	
Zero Reading.	**Other Reading.**	**Zero Reading.**	**Other Reading.**

Mean length, **Mean diameter,**

Volume (give formula and calculation), **cubic centimetres.**

Exercises: (1) Which dimension is likely to have the greater percentage of error?

(2) Is there a reason for making a greater number of measurements of the diameter than of the length?

(3) Make a plain, exact drawing showing the vernier and scale used in the above measurements.

THE MICROMETER

In its common form, the micrometer consists essentially of an accurately cut screw of definite pitch turning in a fixed metal block. One complete turn of the screw advances its end by an amount equal to the distance between two contiguous threads. A longitudinal scale gives the number of whole turns and the graduations on the head of the screw give the fractions of a turn. The micrometer is an essential device in many instruments of precision, among which are the micrometer caliper, the spherometer, and the dividing engine.

The Micrometer Caliper.—The micrometer caliper (Fig. 10) consists of a metal frame with two parallel arms, one of which is threaded to contain the screw while the other is provided with an adjustable stop. The face of the screw and the face of the stop form exactly opposite parallel planes. The screw head is a hollow cylinder or barrel, which slides over the stem or nut, through which the screw passes, and is turned by means of a milled head. Running lengthwise on the stem is a straight line divided into divisions made to correspond with the pitch of the screw. The edge of the cylinder moves aong the edge of this scale as the screw is turned. Fractions of a turn may be observed by noting the divisions engraved on the beveled edge. In a metric caliper the pitch of the screw is usually made 0.5 millimetre, and if the linear scale be divided to 0.5 millimetre, the screw

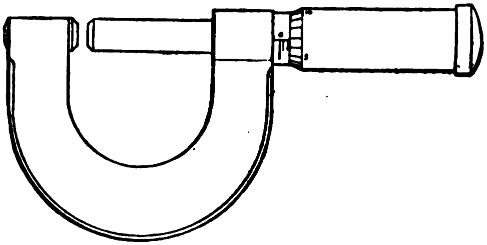

Fig. 10.

will advance 1 millimetre for every two turns. The edge of the cylinder is divided into 50 parts and, therefore, a rotation through a single part will advance the screw 0.01 millimetre. The figures in two sets (1, 2, 3, 4, 5, and 6, 7, 8, 9) sometimes engraved on the beveled edge indicate the tenths of a millimetre. The first set of numbers is to be used when the edge of the cylinder is in the first half of a division, and the second set when it is in the second half. When the end of the screw is in contact with the face of the stop, the zero mark on the beveled edge should coincide with the straight line on the stem, and the edge of the cylinder should coincide with the zero of the linear scale. To aid in setting the screw always with the same pressure approximately, the caliper is sometimes provided with a friction head and ratchet. A certain number of clicks of the ratchet is a fair indication of the pressure desired.

To measure the diameter of a cylinder.

Apparatus: (1) A micrometer caliper. (2) The metal cylinder used in the two preceding experiments.

Errors: (1) The diameter to be measured may not be parallel with the screw. (2) Excessive pressure may spring the metal frame, or affect the dimension to be measured. (3) There may be slight variations in the pitch of the screw.

Procedure: There must be no dust or dirt adhering to the end of the screw or on the face of the stop. Determine the pitch of the screw. Unscrew the head a short distance, noting that at each revolution it passes a division on the stem. Compare the scale on the stem with a scale of known divisions and thus determine the pitch. Having found the pitch, gently rotate the cylinder until the screw and stop are in contact. When the planes touch, the fingers should slip on the milled head in order not to strain the screw by turning too hard. Read the last division visible on the stem and also the division on the cylinder opposite the straight line on the stem. Estimate the tenths of the smallest division on the edge. This completes the zero reading. This reading is to be treated in the manner indicated for the zero of the vernier, and the result marked — when it is below the o of the scale and + when it is above.

Now withdraw the screw far enough to insert the cylinder. Turn the screw until it just touches the cylinder and take care that the diameter lies in a line coinciding with the axis of the screw. Hold the milled head so that the fingers shall slip with the same pressure as in taking the zero reading. Read the scales, estimating the tenths of the smallest division on the cylinder. Make in all eight measurements of the diameter, each time in a different place and direction. Repeat the zero reading before each new trial. Tabulate as follows:

Cylinder No. **Micrometer caliper No.**

Zero Reading.	**Reading (cylinder inserted).**	**Diameter.**

Mean diameter, mm. cm.
Mean diameter found by vernier caliper, cm.

Compute the volume of the cylinder from the length as obtained by the vernier caliper and the diameter obtained by the micrometer. Compare the result with that obtained in the last experiment. Express the volume in cubic centimetres (cc.) in each case.

Exercises: (1) State how many significant figures should be included in the result obtained for the mean diameter, and give the reason.

(2) The length and diameter of a cylindrical solid were found to be 12.343 and 3.286 centimetres respectively. What is its volume if the third decimal in the measurement of its linear dimensions is quite uncertain? Give arithmetical work in full, and retain only the significant figures.

(3) Make a drawing showing the caliper used in the above measurements.

MEASUREMENT OF MASS

It is necessary to distinguish between *mass* and *weight*. The mass of a body is the quantity of matter in it, and is entirely independent of gravity. If a body be taken from one place to another on the earth, or if it could be carried into the depths of space, the quantity of matter it contains, its mass, remains unaltered. The weight of a body, on the other hand, is the force with which the body is pulled downward by gravity. The weight may alter if the body is simply carried from one place to another, since the force with which the earth attracts a given mass varies from place to place even on the surface of the earth. If the earth ceased to attract bodies they would cease to have weight, but their masses would remain precisely the same as before.

The measurement of mass, which is comparison with some standard mass, as the gramme, cannot be accomplished by direct methods. The method in common use is based on the fact that the force with which the earth attracts a body at a given place is proportional to the mass of the body, so that if two bodies are pulled downward by gravity with the same force, their masses are equal. The operation of measuring the mass of a body is called *weighing*, and consists, strictly, in finding how many standard masses are attracted by the earth with a force equal to the attraction on the body which is being weighed. The term "weights" which is so commonly applied to the standards of mass is really a misnomer, yet, by common consent of physicists, the term is retained.

The Balance.—The instrument used for the comparison of masses is the balance (Fig. 11). This is essentially a lever with equal arms, consisting of a light, stiff beam supported at its middle point by a knife-edge which rests upon a plate fixed at the top of a supporting central pillar. At the ends of the beam, at equal distances from the centre, are supported, also by means of knife-edges and small plates, two pans of equal masses in the manner shown in Fig. 12. All knife-edges and plates are accurately ground and are constructed of hardened steel, or sometimes of agate. The bearings are thus rendered almost frictionless, and the lengths of the arms are accurately fixed. The knife-edges are placed in the same

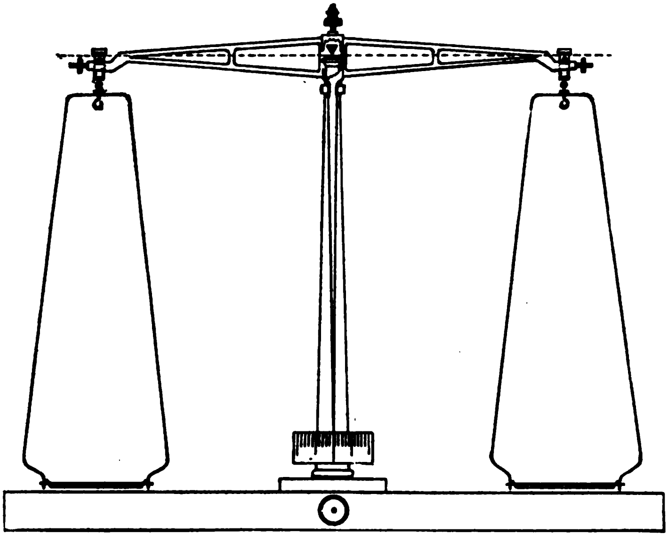

Fig. 11.

horizontal line, and the distance of the centre of mass below the point of suspension is made very small. This arrangement gives sensitiveness of action without losing the stability of equilibrium.

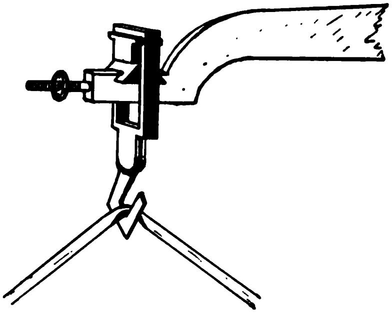

Fig. 12.

A long pointer attached to the beam swings in front of a graduated scale at the base of the pillar, and serves to indicate the deviation of the beam from the position of equilibrium. The use of a long pointer makes it possible to detect very small movements of the beam. If the balance be accurately made and perfectly adjusted, and equal masses placed in the pans, the pointer will remain at rest or will vibrate over an equal number of divisions regularly diminishing on either side of the position of equilibrium. It is found most convenient to call the centre of the scale 10. The divisions are grouped in a series from left to right as follows: 0, 5, 10, 15, 20. By raising or lowering the milled nut which turns on a screw placed over the knife-edge at the centre of the beam, the centre of mass can be raised or lowered and the sensitiveness of the balance changed. At the end of each arm is a small horizontal screw and nut by means of which the beam may be so adjusted that when both pans are empty the pointer is at the centre of the scale. By means of an arrangement operated by turning a large milled head in front of the base of the balance, the beam together with the pans can be raised or released so as to swing freely. A fine balance is provided with a glass case to protect it from the action of dust and currents of air. The case is supported on leveling screws.

The Weights.—The weights are kept in a box, and are so arranged as to allow the correct weight to be quickly found by systematic trial. Heavier weights, made of brass and marked in grammes, are commonly grouped as follows:

50 20 20 10
5 2 2 1

The small weights, made of platinum or aluminium and also usually marked in grammes, include:

.5 .2 .2 .1
.05 .02 .02 .01

The small weights are sometimes marked in milligrammes, thus:

500 200 200 100
50 20 20 10

Sometimes, instead of the sequence 5, 2, 2, 1, we find the sequence 5, 2, 1, 1. If a set is incomplete, the fact should be reported at once. Mixing weights of different sets is a cause of much inconvenience and must be avoided.

Rules to be observed in weighing.

(1) Attend to all details of manipulation, as far as possible, with as great care in the use of a cheap balance as in the use of one of finer construction.

(2) Take a position directly in front of the gradua-

ed scale, so as to avoid parallax in observing the position of the pointer.

(3) In using a balance provided with a case, be careful in raising or lowering the glass front. Do not *throw* it up or down.

(4) See that the balance is leveled; then release the beam and see whether the pointer swings equally on each side of the middle of the scale. If the condition of the balance does not appear to be satisfactory, have the instructor examine and correct the adjustment.

(5) When the balance has once been put in good working condition, it must not be moved, nor must the moving parts of the balance be disturbed.

(6) Always release or arrest the beam slowly—never with a jerk. It is best to stop the vibrations of the beam when the pointer is about to cross the middle of the scale.

(7) Remove dust from the pans with a camel hair brush. Keep the pans clean.

(8) Nothing must be placed on or removed from the pans unless the beam is fixed.

(9) The balance should not be used for comparing masses greater than it is intended to carry. Before a weighing is undertaken, ascertain the maximum load which may be used.

(10) Never completely release the beam when the pointer tends to swing very far to right or left. Only when perfect equilibrium is nearly attained need the beam be released entirely. Large vibrations must be avoided.

(11) Place the body to be weighed in the centre of the left-hand pan and the larger weights in the centre of the right-hand pan, adding the smaller weights in the order of their denomination.

(12) If when the balance is nearly in equilibrium

there is difficulty in getting up a vibration, gently waft the air over one of the pans. While observing the swings of the pointer, the balance case must be closed.

(13) Never handle the weights with the fingers; use the pincers supplied for this purpose. The weights should never be placed anywhere except on the balance pan or in their places in the box.

(14) Substances liable to injure the pans must be weighed in suitable vessels, as weighing bottles or beakers. Volatile substances must be placed in a stoppered bottle.

(15) When a weighing is finished, arrest the beam, remove the body weighed, place the weights in their places in the box, and close the case.

To determine the zero of the balance.

Apparatus: A balance provided with a pointer and scale, and sensitive to 0.01 gramme.

Errors: (1) The point read on the scale may not be the one directly back of the pointer at its turning-point. (2) A vibration may be disturbed after once begun.

Procedure: When the balance is perfectly adjusted, and when the pans are empty and the beam free, the end of the pointer will be in front of the middle division (10) of the scale. This condition is never exactly realized in practice, nor is it strictly necessary. In general, the true resting-point will be found somewhere near the middle division. Furthermore, since time is consumed in waiting for the pointer to come to rest, it is best to determine the resting-point, or true zero, in advance by the method of vibrations; that is to say, by observing the turning-points for three, five, or seven successive swings. A little consideration will show that, if an odd number of swings are taken so that the last swing is on the same side as the first, the point half-way between the mean of all the left-hand and the mean of all the right-hand readings is the point of equilibrium. With the pans empty set the beam oscillating. Close the case and note the extreme posi-

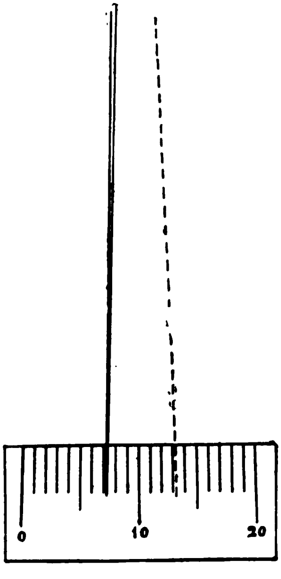

Fig. 13.

tions, or turning-points, of the pointer for five consecutive swings, beginning and ending at the left. A little practice will enable one to estimate the tenths of a division. Record the observations as below:

Left.	**Right.**
5.0	
	13.5
5.5	
	13.0
5.7	
$3 \mid 16.2$	$2 \mid 26.5$
5.4	13.25

$$\text{Zero} = \frac{5.4 + 13.25}{2} = 9.32$$

Repeat the observations twice and record the average of the three trials as the true zero of the balance.

Exercises: (1) If the pans are not hanging quietly on the beam will it interfere with the result? State why and in what way.

(2) Why is it important that the balance be inclosed on all sides in a case while the vibrations are observed?

(3) Make a plain drawing showing what you consider to be the essential parts of a balance.

To determine the mass of a body.

Apparatus: (1) A balance provided with a pointer and scale, and sensitive to 0.01 gramme. (2) A set of metric weights. (3) The metal cylinder used in the first three experiments.

Errors: (1) There is a possibility that the arms of the balance differ slightly in length. (2) Some of the weights may not be accurate. (3) Insufficient care may be taken in the determination of the resting-points.

Procedure: Find the zero of the balance. Arrest the beam and place the cylinder on the left-hand pan. In placing the weights on the right-hand pan, proceed in a methodical manner. Do not try weights at random. The first weights tried should be such as are estimated sufficient to counterbalance the body. Suppose that the first trial weights used are 20 and 10 grammes, and that on slightly releasing the beam, the pointer is seen to move sharply to the left, indicating that 30 grammes is too much. Arrest the beam, remove the 10 grammes, and replace it by 5 grammes. On releasing the beam, the pointer moves to the right, indicating that 25 grammes is not enough. The remaining steps of the process may be indicated briefly as follows: The next weight, 2 grammes, is added and found too much; it is therefore replaced by 1 gramme, which is found not enough. Proceed in this way, first with the decigrammes and then with the centigrammes, trying the weights in descending order, and removing a weight from the pan only when it is found to be too much. Each weight removed should be placed in its proper place in the box. Suppose, now, it is found that with 26.38 grammes the pointer moves to the right of the zero, indicating too small a weight, and that with 26.39 grammes the motion is to the left of the zero, indicating too large a weight. The movement of the pointer to either side of the middle of the scale should not be greater than 5 divisions. If the balance be more sensitive, it may be necessary to add milligrammes and complete the process after the manner about to be described. Leaving the weights on the pan equal to the smaller number, in this case 26.38 grammes, close the balance case, release the beam and determine the resting-point by observing the vibrations. Suppose the resting-point to be 11.28. Add to the weights in the pan 0.01 gramme and redetermine the resting-point. Suppose this to be 9.14. The addition of 0.01 gramme has moved the resting-point from 11.28 to 9.14, that is, 2.14 divisions. This may be called the sensitiveness of the balance for the given load. Arrest the beam and remove the cylinder and weights from the pans. Then redetermine the zero of the balance. Suppose the zero as found at the beginning is 9.30 and at the end, 9.34. The mean of these two results, which is 9.32, will be used. It is now required to find by a simple calculation what weight must be added to 26.38 grammes in order to move the resting-point from 11.28 to 9.32, that is, 1.96 division. This may be illustrated by the following example, which also indicates a form for recording results:

Cylinder No. 8. Balance No. 3. Set of Weights No. 3.

Zero found at beginning, 9.30; at end,	9.34	
True zero of the balance, - -	9.32	
Resting-point for 26.38 grammes,	11.28	
Resting-point for 26.39 grammes,	9.14	
Deviation for 0.01 gramme, - -	2.14	divisions.
Deviation required, - - -	1.96	"

Additional weight required to bring the pointer to the true zero, i. e. from 11.28 to 9.32 equals $\frac{1.96}{2.14}$ of 0.01 gramme, or 0.00915 gramme.

Mass of the cylinder = 26.38 + 0.00915 = 26.38915 grammes.

DENSITY

The *absolute density* of a substance is the mass of matter contained in a unit of volume. If M be the mass of a body and V its volume, its density D is expressed as follows: $D = \frac{M}{V}$. Since density is a concrete quantity, its numerical value in any particular case must depend upon the unit used. Thus in the foot-pound-second system, in which the cubic foot is the unit of volume and the pound the unit of mass, the density of lead is 698.85 pounds per cubic foot; while in the centimetre-gramme-second system, in which the cubic centimetre is the unit of volume and the gramme the unit of mass, its density is 11.2 grammes per cubic centimetre. Since the volume of a body varies with its temperature, the mass of a unit volume will be different at different temperatures. In the case of liquids and gases, especially, it is important to state the temperature to which the density refers.

Find the density of the metal composing the cylinder. Take its mass as found in the experiment just completed, and its volume as computed from the length measured by the vernier caliper and the diameter measured by the micrometer caliper. Arrange the data as follows:

Mass of (here write name of metal) cylinder, gr.
Volume " " " " " " cc.
Density " " " " " gr. per cc.

Show clearly each step in the calculation and retain only the significant figures together with one doubtful one.

Exercises: (1) A block of brass weighing 864.20 grammes has a volume of 102.8 cubic centimetres. Find the density of brass.

(2) What is the diameter of a platinum wire weighing 35 grammes per metre length? The density of platinum is 21.5 grammes per cubic centimetre.

(3) If the density of alcohol be 0.85 gramme per cubic centimetre, what mass of alcohol will be required to fill a bottle whose volume is 683.28 cubic centimetres?

MEASUREMENT OF TIME

A time-measurement frequently required is the period of vibration of a pendulum. A pendulum may be defined as a mass suspended from a fixed point so that it may swing freely. The ideal simple pendulum is one in which the mass is treated as a material particle suspended by a thread without weight. While at rest the pendulum hangs vertically. If drawn to one side, and then released, it goes through a series of vibrations about its position of equilibrium. A single motion in one direction, either to or fro, is called a *simple vibration;* a motion to and fro, a *complete vibration.* The period of a simple vibration is the time of passage from one extreme of the arc of vibration to the other extreme. The period of a complete vibration is the time between a passage through a given point in the arc and the next passage through the same point moving in the same direction.

To determine the period of a pendulum.

Apparatus: (1) A metal ball, about 2 centimetres in diameter, suspended by a fine thread. (2) A metal clamp attached to a firm support about two metres from the floor. Notice the clamp device shown on the laboratory table, a description of which is given in the Appendix. (3) A watch with a seconds-hand. (4) A vernier caliper. (5) A metric scale. (6) A beam compass.

Errors: (1) The pendulum may not be firmly fixed in the clamp. A slight change in length must be avoided. (2) If the minute and seconds hands of the watch do not read even minutes at the same time, some confusion may arise in noting the times of the transits. (3) The central point of the swing may be indefinitely marked.

Procedure: Measure the diameter of the ball with the caliper. Suspend the ball by a fine thread, clamping the upper end of the thread firmly, as in Fig. 14. Adjust the pendulum to a length of about 80 centimetres. Measure the length of the thread from the point, or edge, of suspension to the top of the ball. To this length add the radius of the ball. This will give the true length of the pendulum very nearly. On a cardboard screen, placed behind the pendulum, draw a heavy vertical line, and arrange so that this line is just back of the ball and parallel with the thread when the pendulum is at rest. Calculate the distance that the pendulum must be drawn aside that it may, when released, swing over an arc of not more than six degrees. Since, in any case when the chord of the arc of vibration equals the length of the pendulum the arc is sixty degrees, an arc of six degrees will equal approximately one-tenth of the length. Draw the ball aside about four centimetres and, releasing it, allow it to vibrate. To find the time of vibration, two observers work together, one to count the vibrations and give the signals, the other to observe

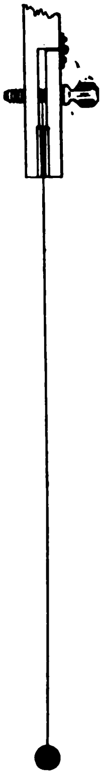

Fig. 14.

the watch and record the times. They should exchange places at the beginning of a new trial. Before beginning the observations, set the minute and seconds hands of the watch to see that they will indicate even minutes at about the same time. In counting the vibrations, the most sharply defined instants are the transits of the pendulum over the vertical line marking the position of equilibrium. For convenience, count the complete vibrations by noting the transits in one direction only, say from left to right. The observer who counts the transits beats time, and at each fifth transit, signifies with a quick tap of the pencil that the time is to be recorded. These times are to be written in hours, minutes, and seconds. The transits whose times are to be noted are indicated by the numbers, 5, 10, 15, 20, and 25, in the first series. Continuing from 25, each transit is counted up to 80. As 80 is approached, the observer calls out the numbers, 78, 79, as a warning, and taps again just as the 80th period is finished. The other observer then resumes taking the times for another series of signals at 80, 85, 90, 95, and 100. It is possible to read the watch correctly to within half a second. Arrange the results as follows:

Diameter of ball, cm.
Distance from support to ball, cm.
Length of pendulum, cm.

Transits,	h.	m.	s.	Transits,	h.	m.	s.
5				80			
10				85			
15				90			
20				95			
25				100			

Average time of 15th, h. m. s.	Average time of 90th, h. m. s.
by 10 and 20,	by 85 and 95,
by 5 and 25,	by 80 and 100,
Time of 15th (direct)	Time of 90th (direct)

Mean time for 15th, Mean time for 90th,
Total time between 15th and 90th, sec.
Number of complete vibrations, 75.
Number of simple vibrations, 150.
Time of one simple vibration, sec.

Include only one doubtful figure in the result. Repeat the experiment with a pendulum about 120 centimetres long. Complete the record of the first trial before beginning the second.

Exercises: (1) From the data recorded find the value of g by the formula given in the text-book.

(2) Does the time of vibration vary with the amplitude?

(3) How does the time vary in relation to the length? Give a simple, concrete illustration.

(4) The length remaining constant, does the time depend upon (*a*) the mass of the vibrating body, (*b*) the weight of the vibrating body?

(5) Two pendulums of equal length are made to vibrate through short arcs whose lengths compare as 5 to 3. Compare their velocities at the lowest point in the arc.

VOLUME OF FLUIDS

Measuring Liquids.—For measuring the volume of liquids various forms of graduated vessels are used. Those most frequently used are shown in Fig. 15.

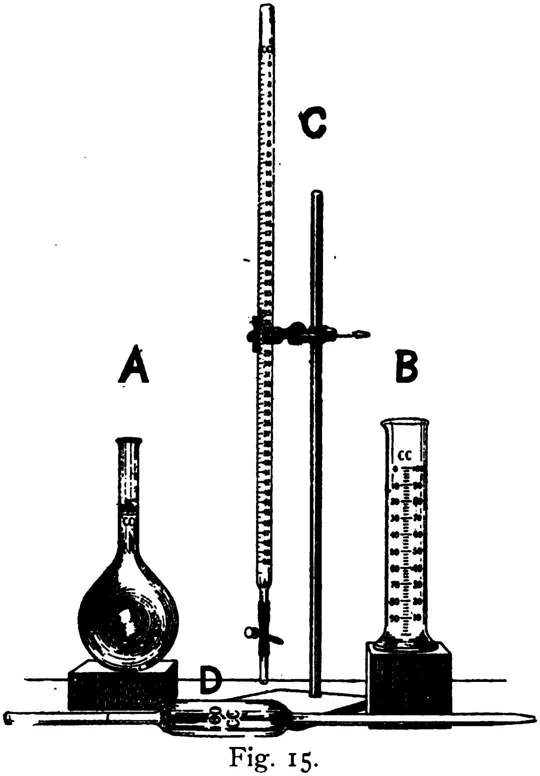

Fig. 15.

The Graduated Flask, A, has a mark etched around its neck. When filled up to this mark, the flask holds a certain volume. The capacity of the flask is marked upon it, together with the temperature at which it is correct. Flasks of one litre, half-litre and quarter-litre capacity are serviceable for most experimental work.

The Graduated Cylinder, B, in its common form, consists of a glass cylinder made to stand upright on a foot. A series of graduations down the side forms the scale. The divisions of the scale are generally whole cubic centimetres, leaving the tenths of a cubic centimetre to be estimated. Two scales are sometimes provided, the one with the 0 at the top used in measuring liquids poured out, the other with the 0 at the bottom used in measuring liquids poured in.

The Burette, C, is used for measuring volumes of liquids with greater accuracy than is possible with either of the above graduated vessels. It consists of a long, slender graduated tube provided with a stop-cock, by means of which the liquid may be run off, varying the flow from the rate of a few drops a minute to that of a rapid stream. The graduations of the scale read to tenths of a cubic centimetre, and extend usually to a 50 cc. mark near the lower end of the tube. The tenths of the smallest divisions can be estimated, thus giving the readings to 0.01 cubic centimetre. While in use the tube is supported vertically by means of a clamp.

The Pipette, D, in its simplest form is a glass tube open at both ends. Pipettes are made of various sizes, such as those which will hold 25, 10, and 5 cubic centimetres when filled to a mark on the tube. The larger sizes have a cylinder or bulb attached to them. The pipette is dipped into the liquid, which is then sucked up the tube to the mark engraved upon it, the top is quickly closed with the finger, and the liquid transferred to another vessel without loss. By slightly releasing the finger, the liquid can be allowed to come out of the tube in drops.

Owing to capillary action, the surface of a liquid contained in a glass tube is not plane, but curved, the extent to which it is curved depending for a given liquid upon the diameter of the tube. This curved surface is called the *meniscus*. Fig. 16 shows the meniscus in the case of water contained in a burette. In reading graduated tubes, it is usual to take the graduation which coincides with the lowest point of the meniscus. Thus, in Fig. 16 the volume of the liquid would be taken as 26.4. In order to obtain the correct reading, it is necessary that the eye of the observer be in the same horizontal plane as the surface of the liquid. By reading from the positions

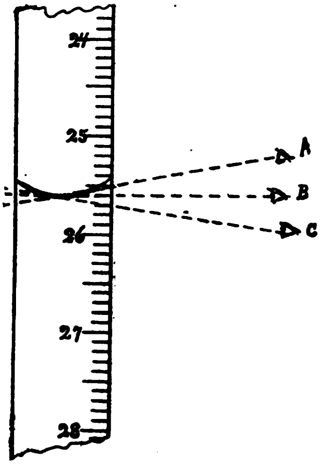

Fig. 16.

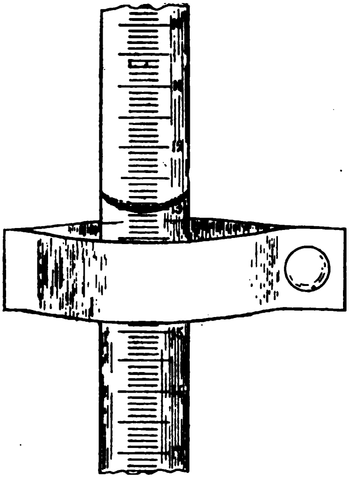

Fig. 17.

A or C instead of from B, the error of parallax easily amounts to one division. When mercury is contained in a tube, it will be found that the meniscus is reversed, curving up at the centre instead of down. In this case the readings should always be taken at the level of the top of the meniscus.

A simple device to ensure that the readings shall be taken from the correct point of observation is shown in Fig. 17. It consists of a strip of stiff paper folded in the middle, with the two ends pinned together with a paper-fastener. This is slipped over the tube, and while it may easily be slid up and down, it will also hold itself in any position in which it may be put. The clip is placed so that its upper edge is just a little below the level of the liquid; then the reading is taken when the eye is in such a position that the back and front edges of the clip coincide and are in the same horizontal plane with the bottom of the meniscus.

Measuring Gases.—The volume of a gas can be measured by collecting it over water or mercury in a vessel graduated into cubic centimetres. Often in exact work it is usual to measure gases in glass tubes over mercury by means of a millimetre scale. Vessels used for measuring gases are called *eudiometers.* Gases expand when warmed, and contract when cooled as liquids do, though to a much greater extent. It is, therefore, necessary to take into account the temperature of the gas. Methods of calculating what the true volume of a gas would be under standard conditions of temperature and pressure will be shown in later experiments.

To determine the internal volume and section of a tube.

Apparatus: (1) A burette, with stand and clamp. (2) A metric scale. (3) A glass tube about 40 centimetres long and 1.5 centimetre internal diameter. The tube is closed at one end. File marks are made across one side of the tube at short intervals apart. (4) A clamp.

Errors: (1) The burette and the tube may not be clamped vertical. (2) Bubbles of air may be delivered from the burette. (3) Readings of the burette may be taken too quickly after running the liquid out. A short time is required for the liquid to run down the inside of the tube and for the meniscus to take a definite shape. (4) There may occur parallax in reading the scales.

Procedure: (I) Secure the tube in a vertical position by means of a clamp. Fill the burette with water up to a point near the 0 mark. To make sure that no air remains in the narrow space between the pinch-cock and the outlet at the bottom, place the upper end of the burette in the mouth and, opening the pinch-cock, blow the water nearly out. Repeat until all air appears to be driven out. Fill the burette again up to a point near the 0 mark. This need not be exact, as the first reading is not necessarily taken at 0. Arrange the burette in position above the tube so that the water may easily be delivered into the latter. Allow the water to flow into the tube slowly, and finally by drops, until the lowest point of the meniscus is opposite the lowest mark of the tube. If the water rises above the mark, the tube must be emptied and dried, and the trial repeated. When a good result is obtained, read the burette. Continue running water into the tube, and taking the burette readings each time that the depression of the meniscus reaches a mark, until the last mark is reached. Pour out the water and place the tube horizontally over a metric scale. Read the position of each mark in succession, beginning at the closed end. From the readings that have been taken may be found the volumes and lengths of the columns of liquid that were run off in succession into the tube. Compute for each column its area of cross-section and its diameter. Tabulate as follows:

Mark.	Burette Read'gs	Vols.	Scale Read'gs.	Lengths.	Areas of Cross-Secs.	Diameters.
1						
2						
Etc.						

Average area of cross-section, sq. cm.
Average diameter of tube, cm.

Measure the entire length of the tube as accurately as possible, and compute (*a*) the internal volume, (*b*) the average volume for each centimetre length, (*c*) the average length for a cubic centimetre volume.

(II) The volume of a solid may be found in a manner briefly indicated as follows: Take the tube used in the preceding experiment, if it is found large enough to contain the body to be measured. For a body take the metal cylinder used in the first experiment. Fill the tube up to one of the upper marks by means of the burette, taking the readings to determine the volume of water run in. Empty and dry the tube and place the cylinder in it. Again run in water from the burette until it reaches the same mark. The difference between the volumes of water run in in the two cases is the volume of the cylinder. Make four trials, measuring the volumes up to a different mark on the tube each time, and tabulate in a suitable form. Average the results obtained and compare this average with the volume as found in the preceding experiments. This experiment illustrates a method commonly employed in determining the volume of an irregular body, *i. e.*, one that cannot be subjected to direct measurement.

Exercises: (1) On what general property of matter does the method of this last determination depend?

(2) State briefly a method for finding (*a*) the volume of a body that floats on water, (*b*) the volume of a body soluble in water, (*c*) the volume of a gas.

(3) A rectangular block displaces 152.295 grammes of water. Its width is 7.1 centimetres, height 3.3 centimetres. Find its length.

(4) What is the interior area of cross-section of a round tube if 3 cubic centimetres of water fills 21 centimetres of it? What is its internal diameter?

HOOKE'S LAW

When forces acting from without are brought to bear upon a body, the equilibrium due to the interaction of the molecular forces between the particles of which the body is composed is disturbed, and the external changes which are brought about may be of two kinds: first, changes of volume, second, changes of form. A change in the form or volume of a body is called a *strain*, while the force acting on a body and producing the strain is called a *stress*. The term *elasticity* is used with reference to that property by virtue of which a body tends to recover its form or volume when released from the action of the disturbing force. A body which recovers completely its original form is said to have elasticity of form or figure; a body which recovers perfectly its former volume is said to have elasticity of volume. Elasticity of volume is possessed in perfection by fluids, which at the time have no elasticity of form. Solids possess both elasticity of form and volume. For every solid there is a limit beyond which, if it be deformed, it no longer returns to its former condition. This is called the *limit of elasticity*. Below this limit the solid body will, on withdrawal of the disturbing force, recover its original form or volume; above this limit a permanent deformation of the body is produced. In a perfectly elastic body, or practically in any solid body within its limits of elasticity, Hooke's Law states that *the strain produced by any force is proportional to that force.*

To verify Hooke's Law by bending a rod.

Apparatus: (1) A wooden rod one metre long and one square centimetre cross-section. (2) Two fulcrums.

each provided with a metal edge. (3) A small pan for suspending weights. The pan is suspended from a metal contact piece which is provided with a binding-post. (4) A micrometer screw mounted on a stand so as to be adjusted to a suitable height. The screw-block carries a binding-post. (5) A set of metric weights. (6) A sensitive galvanometer, with connecting wires. (7) A Leclanchè cell.

Fig. 18 shows the arrangement of the apparatus.

Errors: (1) The fulcrums may be unstable and the metal edges not parallel. (2) The rod may not lie evenly on the metal edges. It should not slip or turn when weights are added. (3) The point of the micrometer screw may not always make proper contact with the metal on the rod.

Procedure: Place the fulcrums at a distance of about 90 centimetres apart, and so that their metal edges are parallel. Lay the rod upon them at right angles to the metal edges. Suspend the scale pan from the middle of the rod. Adjust the micrometer screw so that its point is a short distance above the centre of the contact piece. See that the fulcrums do not rock on the table nor the rod on the metal edges. Join in circuit with the battery, the gal-

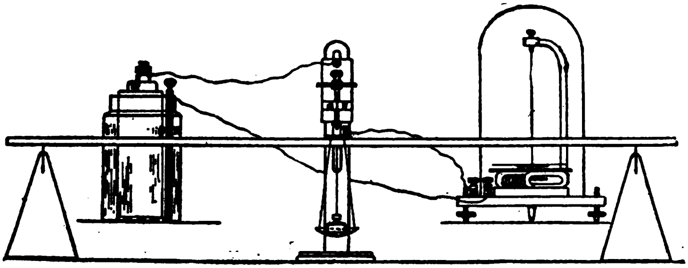

Fig. 18.

vanometer and the micrometer screw. The plane of the galvanometer coil should be parallel with the needle. Turn the screw until it touches the metal piece and note whether a deflection of the needle occurs. The screw should stop turning at the instant a deflection takes place. When a good contact is made, read the micrometer. Place in the centre of the pan 100 grammes, lower the screw as before and take the reading. Turn the screw back to its first position and carefully remove the weight. Again turn the screw down until contact is made and take the reading for the position of the rod without any load. Repeat this process, adding and removing weights and reading the micrometer, using in succession 200, 300, 400, 500 grammes. Tabulate the results as follows:

Weights.	Readings with Weights.	Readings without Weights.	Deflection of Rod.	Deflection per 100 grams.	Ratio of Deflection to Weight.

Average deflection for 100 grammes,

Exercises: (1) State the relation between the weight and the flexure. To what extent will this relation be true?

(2) If a weight of 200 grammes produces a bending of 0.32 centimetre, what will be the amount of bending by a weight of 500 grammes?

(3) Taking weights and deflections as co-ordinates, represent their relation graphically.

(4) If a weight of 1 ton bends a girder half an inch, how much would a weight of 10 tons bend the girder, if Hooke's Law holds true?

To calibrate a Jolly's balance.

Apparatus: (1) A Jolly's balance. (See Fig. 19.) This apparatus consists essentially of a long, fine spiral spring suspended in front of a mirror millimetre scale. A small scale-pan is attached to the lower end of the spiral. From this pan there is suspended by fine wire a similar pan to be used in determinations of specific gravity. At the lower end of the spiral is a short pointer or index, the point of which extends over the scale while the other end moves in front of the mirror. The arm which supports the spring is carried by a rod which slides in the standard of the apparatus and may be clamped at any desired height. There is provided also a platform which may be

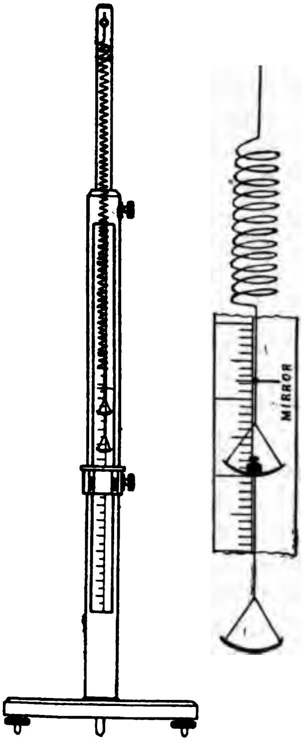

Fig. 19.

raised to different positions in front of the scale. (2) A set of metric weights. (3) A pair of weight-lifters.

Errors: (1) The exact position of the pointer may not be taken on account of parallax. (2) The spiral may be strained beyond its limit of elasticity.

Procedure: In reading the position of the index there is likely to occur the error due to parallax, or the displacement of the index due to different positions of the eye above or below the true position in front of the scale. The error is avoided by taking the readings when the eye is placed so that the index exactly covers its own image in the mirror. The readings are to be expressed in centimetres, estimating the 0.01 centimetre when the index is at rest. First read the position of the index when the pans are empty. This may be called the zero reading. With the weight-lifters, place a small weight, say 0.5 gramme, in either pan and again read the position of the index. Repeat the observations, using in succession weights of 1, 1.5, 2, 2.5, 3, 3.5, 4 grammes, etc., always noting the position of the index both before and after a weight is used. Raise or lower the spring a short distance to change the zero reading and repeat the observations with the same weights. A different series of weights may be used in both sets of observations, if those named are found to be either too large or too small in relation to the sensitiveness of the spring. Tabulate the results obtained in both sets of observations as follows:

Jolly's Balance No.

Weights (W)	Readings without Weights.	Readings with Weights.	Total Elongations (E)	Successive Increments.	$\frac{E}{W}$

What quantities included among the above results are stresses and what are strains?

State the relation between the elongation of the spiral and the stretching force.

Does it appear that the elasticity of the spiral is perfect? Give the reason for the answer.

State the relation between the force of elasticity and the elongation; also the relation between the elasticity of the spiral and the stretching force.

State the relation between the strain and the stress (Hooke's Law).

Up to what limit are the relations as above stated true?

Exercises: (1) From the results obtained in the experiment, find how many grammes will produce an elongation of 1 centimetre in the spiral used. Express the weight also in dynes.

(2) A body placed in the pan gives an elongation of 14.27 centimetres. Express its weight in grammes and in dynes. What is its mass?

(3) Upon what principle, or relation, can Jolly's balance be used in finding the mass or "weight" of a body? Give an illustration.

(4) Taking elongations and weights as co-ordinates, represent their relation graphically by means of a curve.

(5) Make a drawing showing the essential parts of Jolly's balance.

ACCELERATION DUE TO GRAVITY

The motion of a body falling freely through space is plainly accelerated, yet, on account of the high speed acquired in a short interval of time, the exact laws of the motion are not obvious.

The first accurate investigations of this subject were made in 1604 by Galileo, who made use of the leaning tower of Pisa for his experiments. By dropping various bodies from the top of the tower to the ground, and noting the time taken in falling, he succeeded fairly in demonstrating the truth of his assertion, that all bodies, if unimpeded, fall at the same rate. The fact that the resistance of the air is proportional to the area of the surface of a falling body is sufficient to account for certain apparent exceptions, as, for example, the case of a paper or feather falling more slowly than a bullet.

Galileo also investigated the relations between the time of falling, the velocity acquired, and the space described, assuming that the acceleration is constant.

Let v denote the velocity acquired in t seconds, and g the acceleration; then, in general,

$$v = gt. \qquad (1)$$

That is to say, *the velocity is directly proportional to the time spent in acquiring it.*

Also, let s denote the space described in t seconds. Since the acceleration is uniform, the mean velocity acquired at the end of t seconds falling from rest is $\frac{0+gt}{2}$, or $\frac{gt}{2}$. Hence, the body may be treated as moving *uniformly* during t seconds with the mean velocity $\frac{gt}{2}$.

Then $$s = \frac{gt}{2} \times t = \frac{1}{2}gt^2. \qquad (2)$$

Since $\frac{1}{2}g$ is constant, it follows that *the space described is proportional to the square of the time.*

By eliminating t from equations (1) and (2), the relation between v and s can be obtained.

From equation (1), $t = \frac{v}{g}$.

Substituting this value of t in equation (2),

$$s = \frac{v^2}{2g}, \qquad (3)$$

and also, $v^2 = 2gs$.

Therefore, *the space described is proportional to the square of the velocity.*

By making $t = 1$ in the formula $s = \frac{1}{2}gt^2$, we obtain $g = 2s$, or $s = \frac{1}{2}g$.

From which it appears that the acceleration is equal to *twice* the space described in the first second by a body falling from rest.

Galileo found, by careful experiments, that bodies moving under the attractive force of the earth actually obey these laws.

It is a very difficult experimental problem to measure accurately velocities as high as those acquired by falling bodies. On the other hand, it is possible to measure with considerable accuracy the distances through which a body falls in successive intervals of time. One of the direct methods frequently devised employs a tuning-fork of known pitch as a means of obtaining the time intervals and the space passed over during each interval. The apparatus is so arranged that a strip of smoked glass may fall freely in front of a vibrating tuning-fork. To either prong of the fork is attached a stylus which is so adjusted as to trace a sinuous curve on the glass as it falls. From measurements of the waves of this curve the acceleration due to gravity may be computed; and the ways in which the time of falling, the space described, and the velocity acquired are related can be determined.

To determine whether a falling body obeys the laws of accelerated motion.

Apparatus: (1) A wooden base and standard (Fig. 20). On the base is an adjustable post and clamp to hold a tuning-fork. The standard carries an extension-rod, at the top of which is provided a metal pin to hold and release a weighted board. The board is weighted with lead and made so as to fall freely in the groove in front of the standard. (2) A strip of glass to be fastened in front of the board by means of clips. (3) A heavy tuning-fork of definite pitch—say 256 vibrations per second. (4) A metric scale and pair of dividers. (5) Violin bow. (6) A paraffin candle, or Bunsen burner.

Errors: (1) The board in its fall may encounter slight friction in the sides of the groove. (2) If the stylus is not quite flexible it may cause friction, while the motion of its free end may not be quite the same as that of the prong. (3) The rate of vibration of the fork is not exactly the same as it would be without the stylus. (4) The layer of soot on the glass may be too thick. (5) In measuring the length of a group of waves, the points between which the waves lie may be inaccurately determined.

Procedure: Attach a thin stylus of bristle or wire to one of the prongs of the fork by means of soft wax. The stylus should be firmly attached to the prong and with as small a quantity of wax as possible. Coat the glass strip with a thin uniform layer of soot by moving it

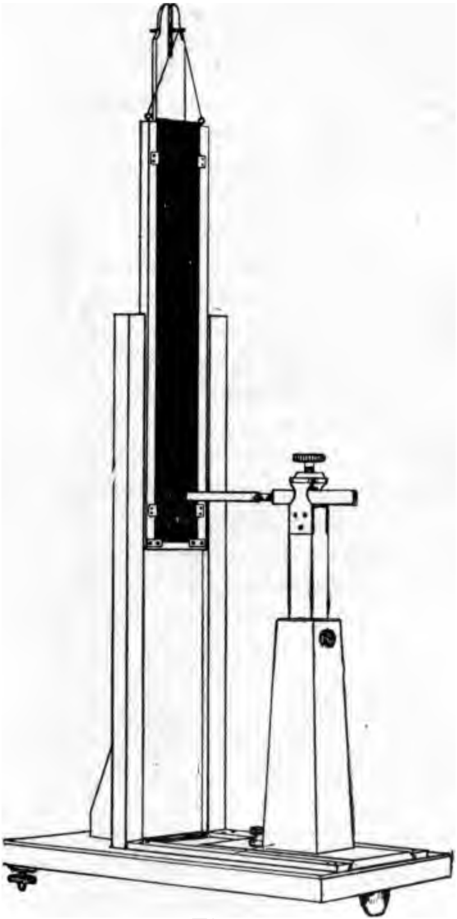

Fig. 20.

slowly to and fro over a smoky flame. Attach the glass to the board and suspend the latter by means of a thread from the releasing pin at the top of the rod. Raise the rod to such a height that the distance through which the board is to fall is equal to the full length of the smoked surface. The apparatus should be leveled so that the board hangs free inside the groove of the standard and, in falling, will be least likely to encounter friction. A heavy weight placed on the base will add to the stability of the apparatus. Adjust the fork so that the stylus is just in contact with the lower edge of the glass and will trace a distinct curve when the board is allowed to fall. Several trials may be necessary before the adjustment is found satisfactory. Clamp the fork firmly in position; then apply the violin-bow, drawing it quickly and with a steady pressure over the edges of the prongs. When the fork is heard to sound loudly, quickly release the board. If the experiment has been successfully performed, there will appear on the smoked surface a curve extending unbroken from end to end of the glass. Repeat the experiment, if necessary, in order to obtain a good curve. Remove the smoked glass and place it on the table with one end raised by props through an angle of about 45°. If a sheet of white paper be now spread beneath the glass, the waves will be plainly visible. By

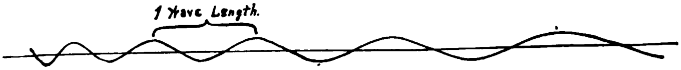

Fig. 21.

a *wave-length* is meant the distance from a given point in the curve to the next point which is in the same relative position. Or, more simply expressed, the wave-lengths are the distances from crest to crest along the curve. Each one of the distances will be the space passed over during a *complete* vibration of the fork. It should also be understood that the vibrations are made in equal times.

In what time did the glass fall the length of any wave?

In what time did it fall the length of any ten waves?

How do the waves vary in length?

What does the whole series of waves illustrate?

Starting at the middle point in the crest of a short wave near the beginning of the curve, measure the groups of ten waves each in succession, as far as the grouping can be carried. Do this each time by adjusting the dividers so that its points lie as nearly as possible *over* the points of beginning and ending of the group, then apply the dividers to a millimetre scale to find the length of the ten waves. Estimate carefully the tenth of a millimetre. Record the results of the measurements temporarily in a column and subtract the first from the second, the second from the third, and so on, to obtain the successive increments. The mean of these differences will be the acceleration *a* per time interval. As the object of the experiment is to determine whether the laws of accelerated motion apply to a falling body, it will be fair in beginning to call the length of the first ten waves, starting from rest, equal to one-half the acceleration. The true point of ending for the first ten waves is nearest found when the measurement of the second group of ten comes nearest to being three times the space passed over in the first time interval, *i. e.* 3/2 of the acceleration. The first three or four waves may not be traced on the glass or may be so much crowded together as to be indistinguishable. A good beginning will usually be made if the first crest at the edge of the glass be counted the end of the fifth wave. If a second trial is necessary count the waves on the lower side of the curve. When the point of beginning of the second group of ten waves has been located as accurately as possible, continue measuring the whole series by groups of ten waves each to the other end of the glass. Avoid blurring the waves with the points of the dividers or rubbing the smoked surface with the hand. Tabulate the results as follows:

No. Vibrations per sec.=			Interval for 10 Waves = sec.				
Successive Spaces, 10 Waves each.	Successive Increments, Accelerations (a)	Total Spaces (s)	Total Times (t)	$\frac{s}{t^2}$	Values of g, $\frac{2s}{t^2}$	Velocities, $a \times t$	$\frac{s}{v^2}$

Average of successive increments (a),

Acceleration due to gravity (g) $= \dfrac{a}{\left\{\frac{10}{256}\right\}^2} =$

Which quantity among the tabulated results appears to be a constant?

What is the significance of the numbers recorded in the columns headed $\frac{s}{t^2}$ and $\frac{s}{v^2}$?

Compare the velocity at the end of any time interval, say the fifth, with the velocity at the end of the first interval.

Compare the total space traversed at the end of any time interval with the space traversed during the first interval.

Compare the space traversed during any single interval with the space traversed during the first interval.

Suppose the waves had been measured in groups of fifteen instead of ten, how would the results in general compare with those obtained?

Taking the space passed over in the first time-interval as unit space, find in whole numbers the values that most nearly represent the relative spaces passed over in the succeeding time-intervals. In the same way represent the relative values of the total spaces s passed over in total times t. Represent also the successive increments a and find the velocity acquired at the end of each interval by the formula $v = at$. Simplify the results obtained by tabulating as follows:

Total Times (t)	Total Spaces (s)	Space each Interval.	Increments or Accelerations (a)	Velocities. $v = at$.	$\frac{s}{t^2}$	$\frac{s}{v^2}$

Exercises: (1) How long must a body fall in order to acquire a velocity of 1 kilometre per second?

(2) Find the distance through which a body must have fallen to acquire a velocity of 1 kilometre per second.

(3) How far will a body fall in the eleventh second of its fall?

(4) During what second will a falling body move over 44.1 metres?

(5) How high is a bridge if a stone dropped from it takes 3 seconds to reach the water?

(6) A stone let fall from a window 20 metres high will strike the ground with what speed?

CONSERVATION OF LINEAR MOMENTUM

To have any change in the state of rest or motion of a body, the presence of a second body is necessary. Take the case of two inelastic bodies of masses M and M′ respectively, of which the first moves with a linear velocity v, while the second is at rest. If the moving body, whose momentum is Mv, strike the other, it will divide its momentum with that other, that is to say, it will travel more slowly after impact, while the other is set in motion; but the two will travel together with a common velocity V in the original direction of the mass M. The combined mass moving with this new velocity V is M + M′; its momentum (M + M′)V is equal to the original momentum Mv. Hence, the velocity V may be found as follows:

$$(M + M')V = Mv,$$

and

$$V = \frac{Mv}{M + M'}.$$

If two inelastic bodies, whose masses are M and M′, are moving with the linear velocities v and v', and if they are free from all external influences, the equation $Mv + M'v' = 0$ (or $Mv = - M'v'$) remains unchanged. (It is understood that if two velocities are in opposite directions, one is +, the other is —). Let the masses move together after impact with the velocity V; then the mo-

mentum of the **conjoined** mass after impact is $(M+M')V$.
Hence

$$Mv + M'v' = (M + M')V.$$

To verify the principle of conservation of linear momentum.

Apparatus: (1) An impact apparatus (Fig. 22).

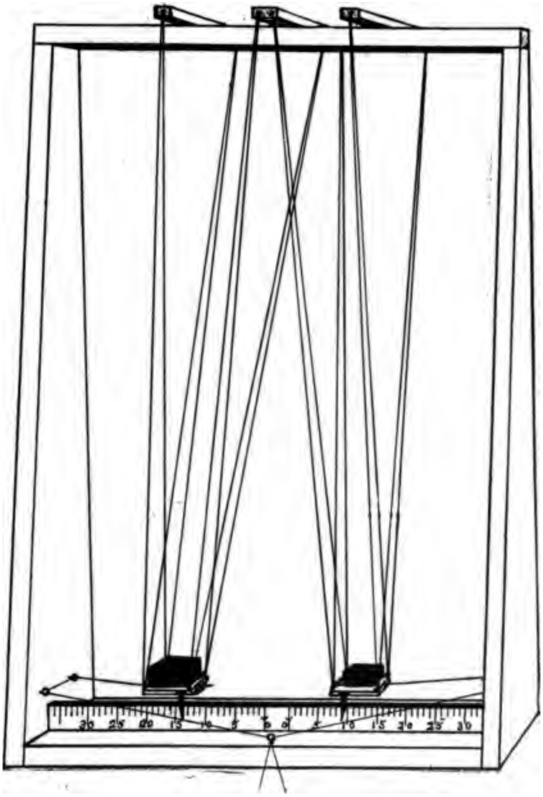

Fig 22.

Two light wooden platforms are suspended by strong threads from cross-pieces at the top of the framework in the manner shown.

The lengths of the threads and the positions of the cross-pieces are so adjusted that when the two platforms are at rest they *just touch,* and when they are pulled apart they always remain horizontal. At the ends of the platforms nearest each other two small blocks of wood are glued. Two sharp metal points project from one of these blocks, so that when the platforms come in contact these points imbed themselves in the corresponding block on the other platform, and thus, after impact, the two adhere together. The platforms can be drawn aside by means of two fine threads which pass through two eyes at either end, and are then brought through an eye at the middle of the base of the frame-work. The threads can be held between the finger and thumb of either hand, and the two pendulums released at the same instant. The positions of the pendulums are read off on a scale, two pointers fixed to the platforms serving for this purpose. If the laboratory table is provided with a cross-beam extending about three feet above the top, the above arrangement can easily be set up. (2) A set of heavy weights, including 1, 1, 2, and 3 kilogrammes respectively.

Errors: (1) The line of motion may be too far from horizontal. (2) The platforms may not be suspended so that they swing in arcs of equal radii. (3) Neither platform must be allowed to rub against the scale at any point. (4) If the platforms are very light their masses need not be considered.

Procedure: From the observations made on the simple pendulum, it was shown that the period of vibration for a pendulum of given length is the same whatever the amplitude, so long as this is not too great. From this it is easily deduced that the velocity with which either platform strikes the other will be proportioned to the distance through which it has been displaced. Hence, the displacements may be taken as representing the velocities of the platforms at the instant of impact. Also, it may safely be assumed that the period of vibration is independent of the mass.

(I) Place a kilogramme weight in each of the platforms, and, drawing them aside through equal distances, let them go. Note the position at which they meet, also the effect at the time of impact. Compare the momenta.

Now displace one pendulum through five centimetres and the other through ten centimetres. Note the point at which they meet, also the effect. What will express the velocity acquired by the combined masses after impact? Express also the momentum. Repeat the trial with displacements of ten and twenty centimetres, repectively.

Place two kilogrammes on one of the platforms, and displacing it each time five centimetres, adjust the displacement of the other pendulum till when they meet they are brought exactly to rest. Compare the momenta. Repeat the trials several times, varying the conditions at each trial.

Enter results as follows:

Left Pendulum.			Right Pendulum.		
Mass.	Displacement or Velocity.	Momentum at Time of Impact.	Mass.	Displacement or Velocity.	Momentum at Time of Impact.

(II) Measure the mass of a given body by means of the impact apparatus. Place the body on one platform and a convenient known mass on the other. Adjust the displacements till when the platforms collide they are both brought to rest. Let d and d' be the displacements of the body and the known mass respectively, v and v', the velocities at the moment of impact; let M be the mass required and m, the known mass used. Then

$$\frac{\text{Momentum of body}}{\text{Momentum of known mass}} = \frac{Mv}{mv'} = \frac{Md}{md'}.$$

Therefore, since the momenta are equal,

$$Md = md',$$

and $$M = \frac{d'}{d} \times m \text{ grammes.}$$

Repeat, using different values for the displacements, and tabulate the results in a suitable form.

Exercises: (1) If a body whose mass is 500 grammes, moving with a speed of 1000 centimetres per second, meets a body whose mass is 100 grammes, moving in the opposite direction with a speed of 500 centimetres per second, what will be the speed after impact, both bodies being perfectly inelastic?

(2) "If a shell explode, its fragments form a system of bodies moving at different velocities."

Explain fully the above statement.

(3) If the motion of the moon in its orbit were to cease, the earth and moon would approach each other. The mass of the earth is about 80 times that of the moon. What part of the whole distance between them would the moon move before collision?

(4) Explain the recoil of a gun against the shoulder. Which has the greater momentum, the bullet or the gun from which it s shot?

MEASUREMENT OF FORCE

When a body which is at rest is set in motion, or when the motion of a body is accelerated or changed in direction, the effect is commonly attributed to a *force.* When a given force is said to act upon a body, causing motion or change of motion, it must be borne in mind that force is not in itself a physical entity and can never be measured unless the mass acted upon and the acceleration produced are known.

When the velocity imparted to a unit of mass in a unit of time (that is to say, the acceleration) is unity, the product $ma=F$ is also unity. In general, a *unit of force* is defined as that observed when a unit of mass is found to acquire unit velocity in one second. If the unit of mass is one gramme, and the unit of velocity is one centimetre per second, the unit of force is called a *dyne.*

The force with which any body is drawn down towards the earth is called the *weight* of that body. It has been found that a body acquires a uniform acceleration when free to fall under the action of gravity. If the gravitational acceleration be represented by g and the mass of the body by m, the weight of the body is expressed by mg.

Since gravity is a force which is always available, it is convenient to use it as a means of measuring other forces. That is,any given force can be said to be equal to the weight of so many units of mass at a certain definite place. Consider a mass placed in the pan of the

Jolly's balance: the force exerted by the spring in holding up the mass is equal to the force exerted by the earth in pulling the mass down; that is to say, the force is equal to the weight of the mass.

The Dynamometer.—An instrument used to measure balanced forces by the strains of a spiral steel spring is called a *dynamometer*. A form of dynamometer suitable for experimental work is shown in Fig. 23. The steel spring is fixed at one end to the end of a block of wood and lies loosely in a groove in the block. At the free end of the spring is a small hook to which is attached an index. On the side of the groove is a scale which may be calibrated so as to read in grammes. It is possible to select a spring of such stiffness that a millimetre space on the scale shall correspond very nearly to a force of one gramme; hence if the total length of the scale be made 20 centimetres, the dynamometer can be used for measuring forces as high as 200 grammes. In using the dynamometer care must be taken to hold it in such a position that the force it is intended to measure acts parallel with the groove. As the spring is likely to acquire a permanent change in length if stretched too far, it must never be under a strain sufficient to stretch it beyond the lowest division of the scale. The dynamometer can be inproved by fixing the upper end of the spring to a block which can be moved up and down the groove and secured in any desired position by means of a thumbnut. By this means the index can be adjusted to the

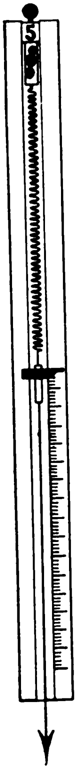
Fig. 23.

zero of the scale before the spring is stretched.

Calibration of a Dynamometer.—The dynamometer can be calibrated by the method employed with the Jolly's balance. Clamp the dynamometer so that the spring hangs vertically, and then suspend by the hook a small scale pan of known weight. Add weights to the pan until the spring is stretched by a force of 10 grammes. Take the reading on the scale opposite the index. Increase the stretching weight 10 grammes at a time, till the lower end of the scale is reached, taking at each step the reading opposite the index. Record the results in a table, giving the readings for the index in one column and the corresponding weights in a second column. Plot the results on a sheet of curve paper, taking the weights as abscissae and the readings on the scale as ordinates. Draw the line through the points, and write the number of the dynamometer near the line. By means of this *calibration curve* the stretching force acting on the spring for any given reading on the graduated scale can be found.

THE PARALLELOGRAM LAW

Let two concurrent forces P and Q act at A (Fig. 24); the force P tending to move the point A in the direction AB, the force Q tending to move the point A in the direction AC. Then, when the two forces act simultaneously, since the point A cannot move in two directions at the same time, it would move under the action of the two forces in *some* direction between AB and AC. Let AD be that direction. A force R acting along AD can be taken of such magnitude that it will produce, when acting alone, the same effect as that produced by the two forces P and Q acting together. This *single* force is then the exact equivalent of the *two* forces P and Q, and is called the *resultant* of the two forces.

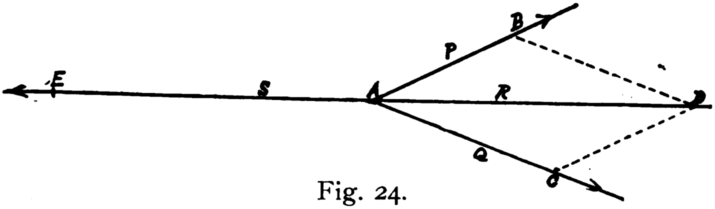

Fig. 24.

When two forces act on a point, their resultant is given by the following proposition called the "parallelogram of forces": *If two forces acting on a point be represented in magnitude and direction by two straight lines drawn from the point, and a parallelogram be constructed on these lines as adjacent sides, then the resultant of the two forces will be represented in magnitude and direction by that diagonal which passes through the point.*

Represent the forces P and Q by straight lines AB and AC (Fig. 24), drawn from the common point of application at A in the directions of P and Q respectively. Also let AB and AC be so drawn that their lengths are proportional respectively to the magnitudes of P and Q. Upon AB and AC as sides construct the parallelogram ABDC; then the diagonal AD exactly represents the magnitude and direction of the resultant R. Produce DA to E, making AE equal to AD; then AE will represent a force S acting at A equal in magnitude to R, but acting in the opposite direction. The two forces R and S exactly neutralize each other, and are in equilibrium; then, since by supposition R produces the same effect as P and Q, the three forces, P, Q, and S will be in equilibrium. Having found the third force S, which together with the two given forces P and Q produces equilibrium, the parallelogram law applied to forces can be verified by construction and measurement.

To verify the parallelogram law.

Apparatus: (1) Three dynamometers. If dynamometers of the kind shown in Fig. 23 are not available, common 8-oz. spring balances can be used. (2) A wooden frame (Fig. 25) about 3 × 3 feet square, on each side of which are holes for the insertion of pegs. (3) A clamp. (4) Some stout thread. (5) A sheet of paper. (6) A metric scale. (7) A rectangular block.

Errors: (1) There is always some friction in a spring balance, and the elasticity of the spring changes with use. (2) Care must be taken to keep the angles between the forces constant during a set of experiments. (3) The zero reading of each balance must be carefully noted, and necessary corrections applied.

Procedure: Place the frame horizontally on the table, and secure it in position by means of the clamp. Take three pieces of stout thread, each about 10 inches long, and tie a small loop at both ends of each thread. Attach an end of each thread to a small ring, about one-quarter inch in diameter, made by bending a wire; then attach the three loose ends of the threads to the hooks of the spring-balances. Place the balances thus connected on the table, passing the rings of two over pegs inserted at any de-

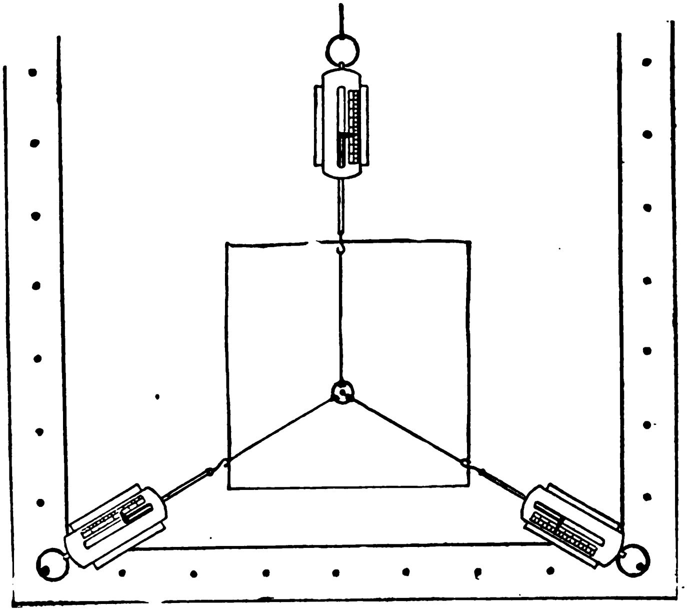

Fig. 25.

sired position in the frame; tie a string to the ring of the third balance. Take the zero reading of each when in the horizontal position, and make a note of the correction to be applied to each in future readings. Now pull the third balance in any desired direction until each balance registers several units, but allowing none of them to be stretched to the full extent; then attach the third balance by means of the string to a peg in the frame. Take the reading of each balance, taking care to apply the proper correction to each.

Place a sheet of paper beneath the threads (Fig. 25), and draw on this paper, just beneath and parallel to each thread, a line a few centimetres in length. Much depends upon the accuracy with which these lines are drawn. It may be necessary to secure the paper by weights or pins while drawing the lines. To draw a line with greater accuracy, place one side of the rectangular block close alongside the thread, taking care not to push the thread out of place, then run the point of the pencil along the edge of the block under the thread. Record alongside each line the value of the force registered by the balance to which the line leads.

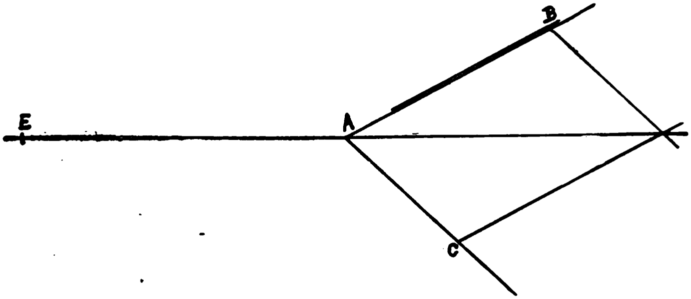

Fig. 26.

Now take a ruler and extend the three lines till they meet at a point. If they do not meet at one point, a new line should be drawn parallel to one of them in such a position as to pass through the intersection of the other two lines. See Fig. 26. This new line is then to be used instead of the original line. Designate the junction of the lines by A, and the direction of each force by an arrow-head. From A measure off the lengths of AB, AC, and AE proportional to the balanced forces. A convenient scale to adopt is 1 centimetre for each ounce, or if the forces are measured in grammes, 1 centimetre for each 10 grammes. Now with AB and AC as two of the sides, draw a parallelogram, taking pains to draw it accurately. Also make a parallelogram with AB and AC as sides, then one with AC and AE as sides. Compare the length and direction of the line AB with the length and direction of the diagonal of the parallelogram drawn on AC and AE; in the same way compare the line AC with the diagonal of the parallelogram on AB and AE, and the line AE with the diagonal of the parallelogram on AB and AC.

What name is applied to the force opposite any one of the balanced forces, considered with reference to the other two forces?

What is any one force called with reference to its two adjacent forces?

State whether the parallelogram law has been verified; if it has been verified, state how.

Exercises: (1) Given two forces represented by lines A and B meeting at a point, make a rule for finding the direction and magnitude of a third force C, which, together with A and B, shall act at the same point and produce equilibrium.

(2) Assume a body of any weight, W, resting on a smooth inclined plane of any slope, such as AB in Fig. 27, prove that the

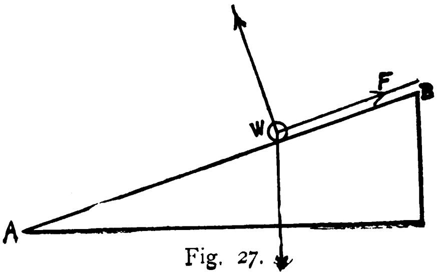

Fig. 27.

force F, parallel to the plane, required to keep the body from slipping down, bears the same proportion to the weight of the body that the height of the plane does to the length.

(3) Show that the following proposition, called the "triangle of forces," is true: *If three forces acting at a point can be represented in magnitude and direction by the three sides of a triangle taken in order, they will be in equilibrium.*

(4) A body whose mass is 100 kilogrammes rests on an inclined plane 3 metres long, one end of which is 50 centimetres higher than the other; what force parallel to the plane will keep it from sliding down? What force parallel to the earth's surface would do the same? Apply the parallelogram law and draw figures.

THE LAW OF MOMENTS

It is a matter of common experience that the effect of a force in turning a body depends on the place at which the force is applied, and on the direction in which the force is exerted. For example, it is easier to shut a door by pushing at the knob than it is to shut it by pushing near the hinges, and easier when the force is exerted perpendicular to the plane of the door than when it is exerted in any other direction.

The effect of a force in producing rotation about a given point is measured by the magnitude of the force multiplied by the perpendicular distance from the point to the line of action of the force. This product of a force into the distance of its line of action from a point is called the *moment of the force* about that point. Thus in Fig. 28, let the irregular outline represent a body. Let F be a force acting on the body and tending to produce rotation about the point O. Then the moment of the force F with respect to the point O is F×OA, where OA is the length of the perpendicular let fall from O upon the line of action of F. It has been found convenient to call the moment of a force producing rotation in one direction

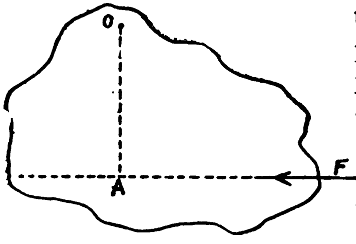

Fig. 28.

positive, and the moment of the force producing rotation in the other direction *negative.* It is generally agreed to call the moment positive, if the rotation tends to be in a direction contrary to that of the hands of a watch. A rotation in the opposite direction would, therefore, be called negative.

The general principle, known as the "law of moments," may be stated as follows: *If a body capable of rotating about a point is in equilibrium under the action of forces in one plane only, the algebraic sum of the moments of the forces is zero;* or in other words, *the sum of the moments having positive signs is equal to the sum of the moments having negative signs.*

Let a balanced disc (Fig. 29) be mounted so as to turn freely about its middle O. Let four forces, A, B, C, D, act at the points M, N, R, S, respectively, in such a manner that the disc remains at rest. The perpendiculars drawn from the point O to the lines in which the forces act are OE, OF, OG, OH, respectively.

The forces A and B tend to turn the disc in one direction, while the forces C and D tend to turn it in the opposite direction. Since the disc is in equilibrium, the combined effect of A and B must balance the combined effect of C and D; or better, the sum of the moments of A and B must be equal to the sum of the moments of C and D, and $A \times OE + B \times OF = C \times OG + D \times OH$.

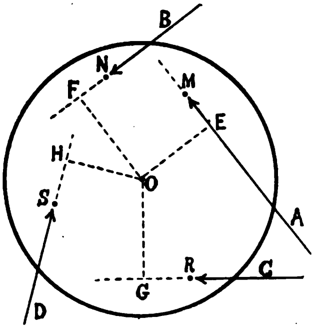

Fig. 29.

To verify the law of moments.

Apparatus: (1) A balanced disc of wood capable of rotating about its centre. The disc is supported on a stand so that its surface lies in a vertical plane. A short distance below the disc is fixed horizontally a millimetre scale. Weights are to be suspended from small pegs to be inserted in holes bored in the disc. See Fig. 30. (2) A set of metric weights. (3) A pair of pans with fine cords for suspension. (4) A platform balance. (5)

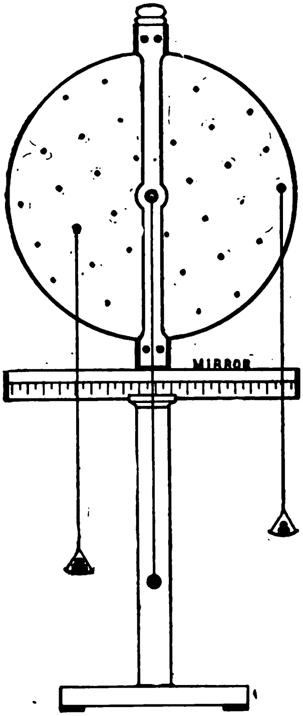

Fig. 30.

An 8-oz. spring-balance. (6) A plumb line; a T-square; a metre rod.

Errors: (1) The bearings of the axle of the disc are not frictionless. (2) The disc may not be perfectly balanced. (3) In reading the positions of the cords in front of the scale, there is apt to be some difficulty on account of parallax. This can be greatly diminished by using a mirror placed along the edge of the scale.

Procedure: Adjust the bearings of the axle of the disc so as to reduce the friction as much as possible. Suspend the plumb line from a point in front of the centre of the axle, allowing it to hang across the scale. Weigh each pan, together with its cord and peg, to the nearest 0.1 gramme.

(I) Place the pegs in holes on the same diameter of the disc and on opposite sides of the centre, and suspend the pans from the pegs. Place weights in the pans until equilibrium is secured. Select the weights from the smaller denominations. Try the effect of rotating the disc to different positions, and note whether the equilibrium is *neutral.* Remove one of the pegs, together with the pan, and place it in a hole so that the line joining the two pegs does not pass through the centre of the disc. Change the weights in the pan, if necessary, and notice that the disc will come to a position of *stable* equilibrium if the line joining the pegs passes under the centre, and that the equilibrium will be *unstable* when the line passes over the centre of the disc.

(II) In the condition of stable equilibrium, read off the scale the positions of the cords and of the plumb line, taking care in the readings to avoid the error of parallax. It may be necessary to tap the disc with the end of a pencil, in order to bring the forces to their proper positions. From the readings obtain the perpendicular distance from the centre of rotation to the line of action of each force. Move the point of application of one of the forces by changing the position of a peg, add weights to those already in the pans, and repeat the observations. Increase the weight in each pan, using the larger masses for the purpose, and take a third set of readings. Make at least five trials in all, each time changing the weights or the relative positions of the pegs.

Tabulate as follows:

Weight of left pan, Weight of right pan,

Forces.		Perpendiculars.		Moments.		Sum of Moments.	$\frac{W}{W'}$	$\frac{p'}{p}$	Errors in per cent.
Left (W)	Right (W′)	Left (p)	Right (p')	Pos. Wp	Neg. $W'p'$	+ or −			

(III) Suspend weights from two points on one side of the centre of the disc and a weight from a point on the other side. Vary the weights or their points of suspension until a condition of stable equilibrium is attained. Take the necessary observations, and arrange the results in such a form as to show that the algebraic sum of the moments of all forces with respect to the axis of rotation is zero. Repeat the experiment, varying the forces or their points of application.

Is it necessary that *all* forces be applied at points below the horizontal line passing through the axis of rotation?

Is the force acting alone on one side of the disc equal to the sum of the two forces acting on the other side?

What is the *essential* condition of equilibrium?

(IV) Suspend a pan and weights from a peg placed in one of the outer holes of the disc, as in Fig. 31. Make the weight at least 200 grammes. In a hole on the opposite side insert another peg. Take a light, strong cord and tie a small loop at each end. Slip one loop over the peg and the other over the spring-balance. Holding the balance steadily in the hand, or better, securing it to a fixed support, adjust it parallel to the surface of the disc and along a line so that a force is applied forming an angle with the vertical. Jar the disc slightly to make sure that the adjustment is good. Then by means of a T-square and a metre scale, measure the perpendicular distance from the line of action of the oblique force to the centre of rotation. Do this by placing one arm of the square along the edge of the scale, the other arm with its edge even with the cord, and sliding the square along the string until the edge of the scale lies directly across the centre of the disc, then take the necessary readings to find the length of the perpendicular. In the usual way measure the length of the perpendicular from the axis of rotation to the line of action of the other force. Make two more trials, each time changing the magnitudes, and relative positions and directions of the forces. If the readings of the balance are given in ounces, they can be reduced to grammes. Tabulate the results in the form given for the observations on parallel forces in (II).

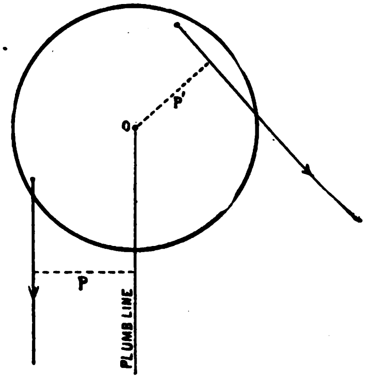

Fig. 31.

Each time that the spring-balance is pulled in a different direction, without at the same time turning the disc, it is observed that the value of the force is changed. Explain.

When a given force tending to turn the disc acts along a given line, can the effect of the force be changed by simply applying the force at a different point on the line? Explain.

Two forces acting in the plane of the disc produce equilibrium about the axis of rotation. Express the relation between the forces in terms of the perpendiculars drawn from the axis to the lines of action of the forces.

Exercises: (1) Two weights, one on each end of a rigid horizontal rod, are balanced on a pivot in such a manner that the equilibrium is neutral. If the rod be tipped, will the equilibrium be destroyed? If the weights bend the rod, what will be the effect?

(2) When a body is subjected to forces in one plane only, what conditions must be fulfilled in order that the body may neither slide nor turn?

(3) If it is wished to upset a tall column by means of a rope of given length pulled from the ground, how can the rope be used to the greatest advantage? Show by means of a diagram.

(4) State a law of equilibrium applicable to all classes of levers.

(5) Applying the law of moments, discuss briefly the theory of the equal arm beam balance used in measuring masses.

(6) When a balance has unequal arms, how can it be used for weighing bodies?

DENSITY OF A LIQUID

When by partial exhaustion of air two liquids are drawn up in vertical tubes which communicate at the top (Fig. 32), the heights of the two columns will be inversely proportional to the densities of the liquids. The pressure of the air on the top of each column is the same. Represent this pressure by p and the pressure of the atmosphere on the free surface of each liquid by A, both measured in dynes per unit area. If h, h' and d, d' represent the heights and densities of the two columns of liquid, each of unit cross-section, their weights will be respectively $h\ d\ g$ and $h'\ d'\ g$. Then at a point in one tube on a level with the surface of the liquid in the cup,

$$A = p + hdg,$$

and at the corresponding point in the other tube

$$A = p + h'd'g;$$

from which it follows, that

$$hd = h'd', \text{ and } \frac{d'}{d} = \frac{h}{h'}.$$

Also, let the heights of the liquids under a certain exhaustion of the air in the tubes be h and h', and under a further exhaustion of air H and H'; then

$$\frac{d'}{d} = \frac{H - h}{H' - h'}, \quad \text{or} \quad d' = \frac{d(H - h)}{H' - h'}.$$

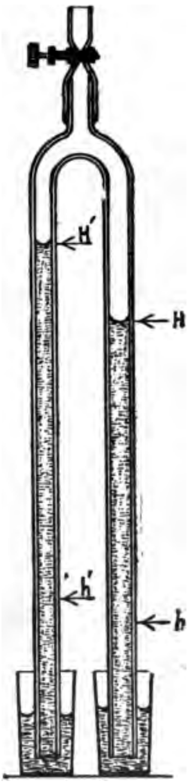

Fig. 32.

From this, if the density of one of the liquids is known, that of the other can be calculated.

To determine the density of a liquid by Hare's method of "balancing columns."

Apparatus: (1) Two glass tubes, each about 80 centimetres long and 1 centimetre in diameter, clamped on a vertical board in such a way as to keep them always parallel and vertical. Behind each tube is fastened a millimetre scale. The two tubes dip into separate cups, each containing one of the liquids whose densities are to be compared. A three-way connector of glass or metal is coupled to the tops of the tubes by means of short rubber joints. (2) A piece of rubber tubing about 50 centimetres long, provided with a pinch-cock and a short glass mouthpiece. (3) A Centigrade thermometer (4) Two small cups. (5) Some liquid whose density is to be determined.

Errors: (1) The tubes may be so small as to cause capillary effects. (2) The scales may not be vertical. (3) Leakages may occur at the joints and at the pinch-cock. (4) In reading the tops of the columns there may occur the error of parallax.

Procedure: Fill one cup with pure water and the other with the liquid whose density is to be determined. Find the temperature of each liquid with a Centigrade thermometer. Keep the thermometer in each cup about a minute before noting the temperature. Open the pinch-cock and, applying the lips to the end of the rubber tube, draw out some of the air so as to raise the liquids to very short distances above the cup. Pinch the rubber tube near the lips, and then close the pinch-cock.

Which liquid appears to have the greater density?

Which column, measured from the free surface in the cup, has the greater weight?

In general terms, what is the magnitude of the pressure that causes each liquid to rise?

Watch the columns to make sure that there is no falling of their surfaces? When assured that all connections are air-tight, read carefully the level of the top of the liquid in each tube. The readings should always be taken at the lowest point of the depression, or meniscus. By placing a small mirror behind the meniscus and at the edge of the scale, the proper position of the eye for avoiding parallax may be found. Draw the liquids further up the tubes, and close the pinch-cock when the taller column reaches nearly half-way to the top. Again take the scale-reading at the top of each column. Repeat the observations four times, raising the columns a very short distance each time before reading the scales. Make the last set of observations with the surface of the taller column reaching nearly to the top of the tube. In each set of observations, subtract from the height of each column as read off on the scale, the height to which it was raised at the beginning. Since the densities of the two liquids are inversely proportional to the heights of the columns,

$$\frac{\text{Density of liquid}}{\text{Density of water}} = \frac{\text{Height of water column}}{\text{Height of liquid column}}.$$

At 4°C. the density of water is 1 (meaning 1 gr. per cc.). If the temperature of water be 4°C., the density of the liquid at its observed temperature will be given simply by the ratio of the heights of the columns. If the temperature of the water be above or below 4°C., the density of the liquid is obtained by multiplying the ratio of the heights of the columns by the density of water at the observed temperature. The density of water at any given temperature can be found by consulting a table of reference. Tabulate the results as follows:

Name of liquid used,

Temp. of water at beginning, ; at end, ; mean,

Temp. of (name liquid) at beginning, ; at end, ; mean,

Top of Column.		Height of Column.		Ratio of Heights.
Water.	Other Liquid.	Water.	Other Liquid.	$\frac{\text{Water Column.}}{\text{Other Liquid Column.}}$

Average ratio of heights of columns (R),
Density of water at mean temperature (d),
Density of (name liquid) = $R \times d =$

Exercises: (1) The tubes used are not necessarily of the same diameter. To what extent may that affect the results obtained?

(2) Is it absolutely important that the tubes be vertical? In what position must each scale always be placed? Is there any reason for keeping the tubes vertical as was done in the experiment?

(3) Is it essential that the tubes be of uniform cross-section?

(4) When the liquids have risen in the tubes, what is included in the total weight supported by the clamp to which the tubes are secured?

(5) Two tubes are joined as in the above experiment. In one tube is a column of mercury (density 13.6), in the other tube is a column of kerosene (density 0.9) measuring 16 centimetres high. How high is the column of mercury?

DENSITY AND SPECIFIC GRAVITY

Density has been defined as the amount of matter contained in a unit of volume. The *relative density* of a substance is the ratio of its mass to the mass of an equal volume of a standard substance. Or, since the amount of matter in a body is proportional to its weight, it is the ratio between the weights of equal volumes of a substance and a standard. This ratio is commonly known as the *specific gravity* of a substance. Specific gravity is usually defined as the ratio of the weight of a given volume of a substance to the weight of an equal volume of pure water at its maximum density. This ratio is evidently the same whatever units are employed. Thus the specific gravity of lead is 11.3, wether the units used be English or metric, or the unit extension in a spring made use of in Jolly's balance. In the C.G.S. system, in which the cubic centimetre is the unit of volume and the gramme the unit of mass, the maximum density of pure water is one gramme per cubic centimetre, at 4°C. Hence, it follows that the numerical value of the density of a substance in the C.G.S. system expresses also the specific gravity of the substance. Thus, in this system, 11.3 expresses numerically both the density and the specific gravity of lead. These relations are not true in the English system. Here the density of water is 62.397 pounds per cubic foot at 4°C. The density of lead is 698.846 pounds per cubic foot, while its specific gravity is written 11.3. From these relations it evidently follows that, if s denotes the specific gravity of a substance, d its density, and D the density of water at an observed temperature,

$$s = \frac{d}{D} \text{ and } d = sD.$$

Also, if M be the mass of a substance whose volume is V, then

$$M = Vd = VsD.$$

If the density of water be 1 gramme per cubic centimetre, the above equations become

$$d = s \text{ and } M = Vs.$$

The principle of Archimedes suggests a method by which the density of a substance may be determined. When a solid is immersed in a liquid, it is buoyed up by a force equal to the weight of the liquid displaced. Or in other words, when a solid is immersed in a liquid, its loss of weight is equal to the weight of a volume of the liquid equal to the volume of the solid. When a solid is weighed first in air, then in a liquid, the difference between the two weights will be the weight of a volume of the liquid equal to that of the solid. The ratio of the weight of the solid in air to the weight of the liquid displaced expresses the specific gravity, or relative density, of the solid. From this, if the density of the liquid at an observed temperature be known, the density of the solid can be calculated by the formula

$$d = sD,$$

in which d is the density of the solid, s the specific gravity, and D the density of the liquid.

To determine the density of a solid and the density of a liquid by means of a Jolly's balance.

Apparatus: (1) A Jolly's balance. (2) A small glass vessel. (3) A Centigrade thermometer. (4) A piece of the substance whose density is to be determined. (5) A quantity of the liquid whose density is to be determined. (6) A glass sinker.

Errors: (1) The lower pan may not be at the same depth in the water throughout the experiment. (2) Air bubbles may cling to the solid. (3) Friction between the pan and the sides of the vessel is to be avoided. (4) The upper pan must be kept dry.

Procedure: (I) Within its limit of elasticity the elongation of the spring has been found to be proportional to the weight tending to stretch it. The elongations of the spring may then be used instead of actual weights in finding the specific gravity of the substance. Place the vessel containing water on the adjustable shelf of the balance and arrange so that the lower scale-pan is completely immersed in the water, as in Fig. 33. By raising or lowering the shelf to correspond with the elongation of the spring, the lower pan can be kept immersed to the same depth throughout the experiment. Determine the temperature of the water. When the pans are at rest take the reading of the index (*a*). Lower the shelf and place the body on the upper pan. Read the position of the index (*b*). Transfer the body to the lower pan under water and again take the index reading (*c*). Take the temperature of the water again.

Express in literal terms (*a, b, c.* above) the weight of the solid in air.

What is the value of the buoyant force upon the lower pan when submerged in water?

What is the total buoyant force when the solid is placed in the lower pan?

What is the buoyant force of the solid alone?

Express in literal terms the weight of the water displaced by the solid.

Express the specific gravity of the solid.

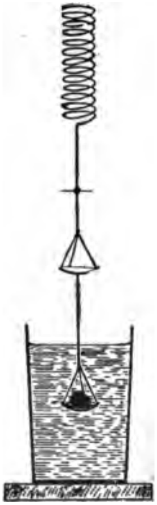

Fig. 33.

Why must the lower pan be kept under water at the same depth throughout the experiment?

Determine the density of the solid in both systems of units, knowing (1) the specific gravity as found by the experiment, (2) the density of water in grammes per cubic centimetre at the observed temperature as given in a table of reference, and (3) that the density of water is also about 62.4 pounds per cubic foot. Tabulate as follows:

Name of substance, Number of Jolly's balance,
Temperature of water at beginning, at end, mean,
Scale reading with pans empty (*a*),
Scale reading with body in upper pan (*b*),
Scale reading with body under water (*c*),
Specific gravity of the solid,
Density of water at observed mean temperature, gr. per cc.
Density of (write name of solid) grammes per cubic centimetre.
pounds per cubic foot.

(II) The same general method may be used to determine the density of a liquid. Attach the glass sinker to the end of the spring and take the reading of the index (*a*). Place the vessel containing water underneath the sinker so that the latter is completely immersed. Read the position of the index (*b*). Find the temperature of the water. Withdraw the sinker from the water, absorb all moisture from its surface by means of blotting paper, and again take the reading of the index (*a*). Place a vessel containing another liquid on the shelf, immerse the sinker, and take the index reading (*c*). Find the temperature of the liquid.

What is the buoyant force on the sinker in water?

Express the weight of the water displaced.

What is the buoyant force on the sinker in the liquid?

Express the weight of the liquid displaced.

Write the literal expression for the specific gravity of the liquid.

Compute the density of the liquid, knowing (1) its specific gravity, (2) the density of water at the observed temperature. Tabulate as follows:

Name of liquid,
Temperature of water,   Temperature of liquid,
Scale reading with sinker in air (*a*),
Scale reading with sinker in water (*b*),
Second reading with sinker in air (*a*),
Scale reading with sinker in the liquid (*c*),
Specific gravity of the liquid,
Density of water at the observed temperature,   gr. per cc.
Density of (name liquid),   grammes per cubic centimetre.

Exercises: (1) Given X as the density of a substance in the metric system, what is its specific gravity in the English system?

(2) A certain substance is found to weigh only one-third as much in water as in air. What is its specific gravity?

(3) A mass of silver weighing 100 grammes is suspended by a string in water. If the density of silver is 10.5, what is the tension in the string?

(4) The same volumes of water and another liquid weigh 1.8 kilogramme and 2.34 kilogrammes respectively. Find the density of the liquid.

(5) A sphere 1 metre in diameter is just immersed under water. What is the force upward? What is the force upward when the sphere is surrounded by atmospheric air at 0°C.?

THE BAROMETER

The measurement of the total atmospheric pressure was first accomplished by Torricelli in 1643. He took a glass tube about a metre long, sealed it at one end, and filled it with mercury. Closing the open end with the finger, he inverted the tube and plunged the lower end beneath the surface of mercury in a basin. On withdrawing the finger, the mercury in the tube fell until its top surface was about 76 centimetres above the mercury surface in the basin. The vacant space above the mercury column has since been known as the Torricellian vacuum. Since no pressure is exerted upon the top of the mercury within the tube, it is inferred that the column is sustained by the pressure of the atmosphere upon the mercury in the basin; in other words, the vertical height of the column above the level of the mercury in the basin indicates the atmospheric pressure. Since, in a liquid, pressure on a given area is transmitted equally in all directions to all equal areas, the pressure due to the weight of the mercury column on the sectional area of the tube, on the same level as the surface of the mercury in the basin, is equal to the pressure of

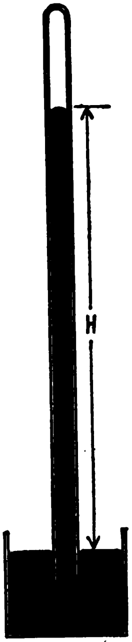

Fig. 34.

the atmosphere on an equal area. Suppose the sectional area of the tube is a square centimetres and the height of the column H centimetres, the pressure on this area is equal to the weight of aH cubic centimetres of mercury. If d be the density of mercury at the given temperature, the pressure on a square centimetres will be aHd grammes, and on one square centimetre, $a\mathrm{H}d/a =$

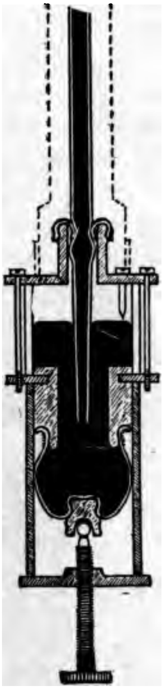

Fig. 35.

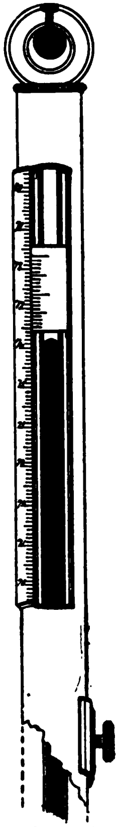

Fig. 36.

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Hd grammes. Multiplying this by g gives a pressure of Hdg dynes on one square centimetre.

Torricelli's apparatus, when provided with a scale for the purpose of reading the height of the column of mercury, is called a *barometer.*

A form of barometer most commonly employed for accurate observations is that of Fortin (Figs. 35 and 36). The glass tube is surrounded by a protecting sheath of metal to the lower end of which the reservoir is attached. The upper portion of the reservoir is made of glass through which the surface of the mercury can be seen. The bottom is made of buckskin, thus allowing an easy adjustment of the height of the mercury by means of a screw from below. An ivory pointer is fixed inside the reservoir, in such a way that its tip marks the level of the zero division of the scale. Near the upper end of the tube a rectangular space permits the observation of the top of the mercury column (Fig. 36). The front edge of this space carries the graduations of the upper portion of the scale. The reading is made by means of a vernier engraved on a movable metal piece which is adjustable by means of a rack and pinion. As the "height of the barometer" is the distance *vertically* from the mercury surface in the cistern to the mercury surface in the tube, this distance is found directly from the scale. The upper part of the scale being the only portion of it that is ever used, the lower part is always omitted.

In the Gay-Lussac barometer, also called the *siphon barometer,* no distinct reservoir is provided. In one of its simplest forms it consists of a bent glass tube, one of the branches of which is much longer than the other (Fig. 37). The longer branch, which is closed at the top, is filled with mercury, as in the Fortin barometer, while the shorter branch, which is open, serves as a cistern. The vertical distance between the free surfaces of the mercury in the two branches measures the atmospheric pressure. The measurement is facilitated by means of an adjustable scale and vernier.

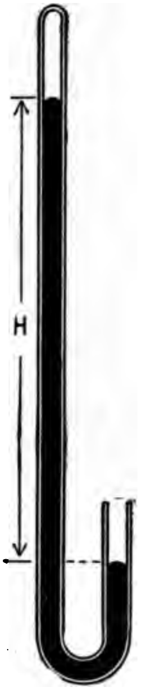

Fig. 37.

To measure the atmospheric pressure.

Apparatus: (1) A barometer. (2) A thermometer attached to the barometer.

Errors: (1) The instrument must be fixed or suspended so that it hangs *vertically.* (2) The top of the mercury in the cistern must be at the zero of the scale. (3) The zero of the vernier must be accurately at the same level as the top of the mercury column. (4) Every barometer has an *instrumental error* peculiar to itself, which may for convenience be considered as including also such causes as capillarity, imperfect vacuum and incorrect scale divisions. The instrumental error should be ascertained by comparison with a standard barometer, and the correction posted in a convenient place in the laboratory.

Procedure: Read the thermometer fixed alongside the barometer, care being taken that the presence of the observer does not introduce an error by raising the temperature of the surrounding air. If the thermometer readings are subject to error, the corrected temperature should be recorded.

Tap the barometer in the vicinity of the upper level of the mercury to overcome the adhesion between the mercury and the glass. Adjust the level of the mercury in the cistern. To do this, first shake the cistern *gently*, then turn the screw until the tip of the pointer touches the mercury so lightly that the depression caused by it is just not visible. Or, watch the reflection of the pointer in the mercury; when the pointer and its image appear to come into contact, the adjustment is complete. Bear in mind that the ivory point is the *zero* of the barometer scale, and from it all measurements are made.

Raise the metal piece on which the vernier is engraved until the lower edge is clearly above the convex surface of the mercury, and then carefully lower it until its edge appears to be just in contact with the top of the meniscus. If the eye be moved slightly up and down, no line of light should appear at the middle.

Read the scale and vernier. Begin at the beginning, making all adjustments over again, and repeat the reading. Repeat until successive readings agree.

In order to reduce the barometer reading to ***standard conditions***, several important corrections are necessary. These may be indicated as follows:

(1) *Correction for temperature.*—Barometer readings are reduced to what they would be if the mercury and scale were at a temperature of 0°C. Since mercury expands 0.00018 of its volume for each degree C., the true reading at zero would be $h = H\ (1 - 0.00018t)$, where H is the observed height and t the temperature. The same rise in temperature has caused the metal scale and tube to expand, so that the observed height is too small. Assuming that the scale is graduated for 0°C., and that the metal of the scale expands 0.000019 of its length for each degree C., as is the case with brass, the

complete correction due to the expansion of both mercury and scale will be

$$h = H[1-(0.00018-0.000019)t].$$

This double correction amounts to 1.86 millimetre if the barometer stands at 760 millimetres and the temperature is 15°C., provided the scale be of a metal whose coefficient of expansion is about that of brass.

(2) *Correction for elevation.*—In addition to the above a correction for elevation must be made, in order to reduce all readings to sea-level. This correction varies with the temperature of out-door air and can be obtained from tables. It is about 9 millimetres for each 100 metres of elevation.

(3) *Correction for latitude.*—The weight of the column of mercury which is balanced by the pressure of the air depends on its position relatively to the earth. For example, for the same atmospheric pressure, the barometer would stand nearly 4 millimetres higher at the equator than at the pole, because the value of gravity is slightly smaller at the former than at the latter place. In order to make readings taken at different places comparable, they must be reduced to what they would be at the standard position, sea-level, in latitude 45 degrees.

(4) *Correction for capillarity.*—On account of the capillary action between the mercury and the glass of the tube, the level of the mercury is depressed. Barometers which are intended for very accurate work have tubes whose diameters are so great that the error due to capillarity is always small. The amount of correction to be applied under certain conditions may be obtained from tables.

Apply the corrections explained under (1), using data given in the table (See Appendix).

The correction for elevation, explained under (2), can be applied providing the necessary tables are at hand. In case such correction is to be made, it will be necessary to find the temperature of the out-door air.

Corrections explained under (3) and (4) should be included in the instrumental error.

What would be the volume of the column of mercury if the area of cross-section were 1 square centimetre?

What would be the weight of such a column if the density of mercury is 13.6 grammes per cubic centimetre?

Name and define in exact terms the pressure which sustains this column of mercury.

Express this pressure in dynes.

What change would occur in the observed height of the mercury column if the area of cross-section were changed?

State in what way the barometer "measures" the atmospheric pressure.

Make observations for several consecutive days at the same hour of the day, applying corrections for temperature (and for elevation, if possible,) in each case. Also record and apply the correction for instrumental error if it be known.

Tabulate as follows:

Date and Hour.	Temperature of Barometer.	Barometer Reading.	Correction for Temperature.	Other Correct'ns. (State kind)	Corrected Height.

Plat the above results in a curve, using dates as abscissae and heights of the barometer as ordinates.

Exercises. (1) What is the height of a water barometer when the mercury barometer reads 760 millimetres? Density of mercury=13.6.

(2) A barometer is used to measure the exhaustion in the receiver of an air-pump. What fraction of the air has been withdrawn when the pressure has fallen from 76 to 30 centimetres?

(3) What is the pressure of the air in grammes upon 1 square centimetre when the barometer stands at 74 centimetres? Calculate the pressure in dynes.

(4) A barometer is provided with a brass scale which is correct at 0° C. The apparent height is 76.3 centimetres and the temperature is 41°. What is the correct height? (See paragraph above, *Correction for temperature.*)

(5) A soap-bubble has a diameter of 10 centimetres. Compute the pressure of the atmosphere upon it.

BOYLE'S LAW

It is a general fact, based on observation, that if the volume of a given quantity of gas is diminished, the pressure which it exerts is increased. A very good illustration is seen in the action of the common pop-gun. When the piston is pushed quickly in, the increase in pressure developed in the compressed air expels the cork plug with a small explosion.

Shortly after the time of Torricelli's experiments, Robert Boyle, one of the founders of the Royal Society of London, became interested in the following question: "Exactly *how* does the volume of a given mass of gas vary when the pressure is varied?" Boyle announced the results of his investigations in his "Defence of the Doctrine touching the Spring and Weight of Air," published in 1662. He showed that within the limits of accuracy allowed by his apparatus, the pressure exerted by a given mass of gas at constant temperature is inversely proportional to the volume which it occupies.

Thus, consider a mass of air enclosed by mercury in the shorter arm of a U-tube as indicated in Figs. 38 and 39. When care is taken to make the level of the mercury the same in the two arms (Fig. 38), the mass of confined air is under atmospheric pressure. Call the volume of the air in the shorter arm V, and the atmospheric pressure P, represented by the height of the barometer at the time, expressed in centimetres. If more mercury is poured into the longer arm, until the difference in the levels of the mercury in the tube represents a pressure p, also expressed in centimetres, the pressure to which the enclosed air (Fig. 39) is now subjected is $P+p$, and the volume of the air is diminished, becoming, say V'. Boyle's Law is then stated as follows:

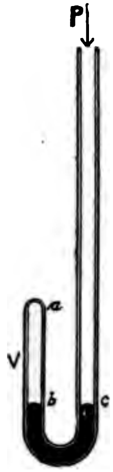

Fig. 38.

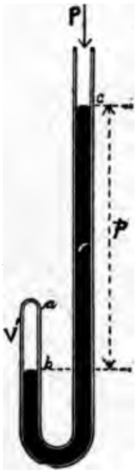

Fig. 39.

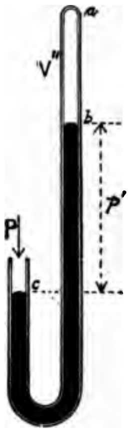

Fig. 40.

$$\frac{V}{V'} = \frac{P+p}{P},$$

or

$$PV = (P+p)V'.$$

Mariotte, a French physicist, announced the same law in 1679, and it was further demonstrated that the law holds for pressures less than one atmosphere. If a suitable tube be provided, and the level of the mercury in the open arm be adjusted so that it is below the level of the mercury in the closed arm, and assuming the same mass of air as in the above example, the pressure to which the air is subjected is $P—p'$, in which p' is the pressure due to the difference in the levels of the mercury (Fig. 40). Calling the volume of the air V'' in this case, Boyle's Law may be written to cover all cases, thus:

$$PV = (P + p)V' = (P - p')V''.$$

Or, if $(P+p)$ and $(P-p')$ be expressed by P' and P'', respectively, the law can be given thus:

$$PV = P'V' = P''V'',$$

which is better written to cover all cases in the following form:

$$PV = K, \text{ a constant.}$$

The importance of the last expression of the law is obvious. Having measured the pressure and corresponding volume of a gas, the constant K can be found. From this it is possible to *compute* the pressure corresponding to any other volume, or, conversely, the volume corresponding to any other pressure.

To determine whether Boyle's Law holds for a given quantity of air.

Apparatus: (1) A closed tube of uniform bore connected with an open tube by means of india-rubber tubing. Both tubes are supported vertically by carriers that can be moved up and down a tall standard, and can be clamped at any desired position. Sufficient mercury is used to fill the rubber tube and a portion, say-one-half, of each glass tube. Between the glass tubes is a vertical millimetre scale, on which the levels of the mercury surfaces in the tubes can be read off. It is of some importance that the end of the closed tube be square, or only slightly curved, otherwise there will be some difficulty in making the observations. The top of this tube can be provided with a metal cap by means of which it can be closed air tight. In filling the tubes with mercury the cap is unscrewed, then screwed on tight again when the mercury levels are properly adjusted. The apparatus is shown in Fig. 41. (2) A barometer.

Errors: (1) When the air is compressed or expanded in the closed tube its temperature may be changed considerably. (2) If the tubes are not clean, the mercury will stick to their sides. (3) The internal cross-section of the closed tube may not be uniform. (4) The quantity of the air under consideration must be kept constant during the experiment.

Procedure: Place the standard of the apparatus at a convenient elevation on a table or other support. At the beginning of the experiment read the barometer. If the barometer readings are given in inches, reduce to centimetres, carrying the result to the second decimal figure. Adjust the tubes in a position near the lower end of the scale, so that the mercury surfaces are at the same level. What is now the pressure on the air in the closed tube? Tap the tubes until the mercury surfaces become convex; then read on the scale (*a*) the position of the top of the inside of the closed tube, (*b*) the highest point of the meniscus in the same tube, and (*c*) the highest point of the meniscus in the open tube. Since the bore of the closed tube is assumed uniform, the volume of air enclosed may be taken as proportional to the length of the tube which it occupies; that is, if the length occupied be doubled, the volume is also doubled.

The volume of the confined air at each stage of the experiment may then be represented by the number of centimetres difference between the first two readings, (*a*) and (*b*), to be taken.

Now move the open tube up a few centimetres, and take the three readings as before. Find the difference between the levels of the mercury in the tubes. What changes have taken place in the air

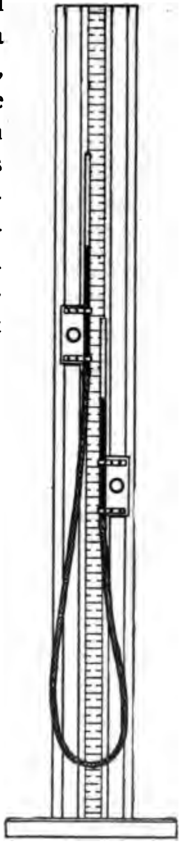

Fig. 41

in the closed tube? What is now the total pressure on the air in the closed tube? Continue raising the open tube, and repeat the observations at positions about 10 centimetres apart until the tube is as high as it can be. Avoid stretching the rubber tube.

Now raise the closed tube to a position near the top of the standard, so that the mercury surfaces will be at about the same level near the top of the scale. Lower the open tube a few centimetres, and again take the readings to obtain the volume of the air and the difference between the levels of the mercury. What is now the pressure on the air in the closed tube? Continue lowering the open tube, and repeat the observations at positions about 10 centimetres apart until the tube is as low as it can be. It will not be necessary to move the closed tube more than once during the experiment. In raising or lowering the tubes, the movement should be steady, so as to avoid possible injury to the apparatus. When all observations have been taken, bring the tubes to the same level again near the lower end of the scale. Tabulate as follows:

Apparatus No. **Barometer reading,**

Closed Tube.		Open Tube.	Volume of Air (V)	Difference in Mercury Levels.	Total Pressure (P)	PV (K)
Top.	Mercury.	Mercury.				

Explain the significance of the numbers recorded in the last column, under the heading PV.

What is the most probable value of the constant (K) for the given mass of air?

In this experiment, what is the stress and what is the strain?

State your conclusion regarding the elasticity of the air.

State the relation between the density of the air and the pressure upon it. Express the relation by an equation, in which D is density.

Plot a curve, taking pressures (P) as abscissæ and volumes (V) as ordinates. How does the curve show a verification of Boyle's Law?

Exercises: (1) At what pressure will one litre of air (at standard pressure) occupy 990 cubic centimetres?

(2) Take the air confined in the tube used in the above experiment; at what pressure could the air be made to occupy a volume measured by one centimetre of the length of the tube?

(3) A solid, close-fitting piston is situated in the middle of a closed cylinder 10 inches long, and there are equal quantities of air on each side of it. The piston is pushed until it is within an inch of one of the ends; compare the pressures on each side.

(4) A bottle is found to contain 2 grammes of air when the barometer stands at 75 centimetres. What mass of air will it contain when the barometer is at 73 centimetres?

VELOCITY OF SOUND IN AIR

If a tuning-fork be held near the open end of a tube, the air column in the tube will be set vibrating and will resound if the note it can emit is the same as that given by the fork. It is proven in text-books that, if the tube be closed at one end, the wave given out has a length w which is approximately four times the length l of the tube, or

$$w = 4l \text{ nearly.} \qquad (1)$$

The relation between the velocity of sound V, the wave-length w, and the vibration frequency n, is also shown to be

$$V = nw. \qquad (2)$$

From equations (1) and (2) is obtained the equation

$$V = 4nl. \qquad (3)$$

Since equation (1) is only approximate, it follows that equation (3) is not strictly true. A correction is necessary for the open end of the tube. It is calculated that this correction may be made by increasing the length of the resonance-column by an amount equal to 0.8 of its radius. Introducing this correction, equation (3) becomes

$$V = 4n(l + 0.8r), \qquad (4)$$

where r is the radius of the tube. This correction is only an approximation and probably has a special value for every diameter of tube.

From the explanations given of the cause of resonance in a tube, it is easily seen that a note will be similarly reinforced by greater lengths of air-columns, and that a difference in length between two successive lengths of the tube which resound to the same note is one-half the wave-length of that particular note. If, then, this difference be represented by L, the equation for the velocity of sound in air may be written

$$V = 2nL.$$

To determine the wave length of a given note and the velocity of sound in air by means of the resonance-tube.

Apparatus: (1) A glass resonance-tube about 150 centimetres long and 5 centimetres in diameter, held by means of a clamp in a vertical position (Fig. 42). The lower end of the tube is closed by means of a cork, or rubber stopper, into which is fitted a short piece of glass tubing. Over the top edge of the tube is hung a short glass U-tube, one arm of which is joined to a length of rubber-tubing, while the other arm opens inside the resonance-tube at a point about 10 centimetres below the top. This device is to be used as a listening tube. (2) A large bottle containing water. Into the neck of the bottle is inserted one arm of a glass tube bent in the form of a siphon. A length of rubber tubing connects the siphon with the short tube at the bottom of the resonance-tube. (3) A metre rod. (4) A tuning-fork of known number of vibrations per second. (5) A thermometer.

Errors: The sound produced by the fork changes in intensity and thus makes it difficult to determine accurately the position of maximum resonance.

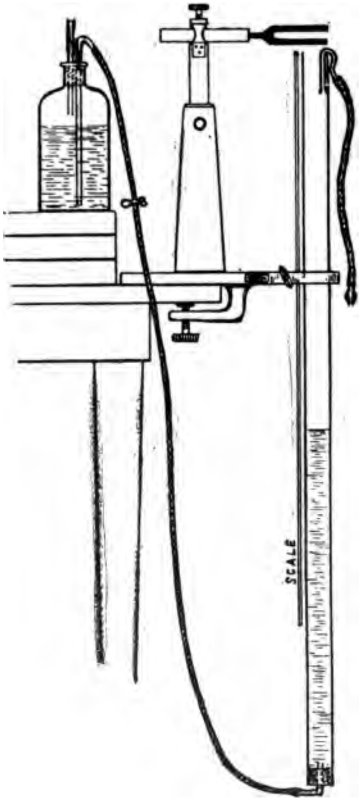

Fig. 42.

Procedure: Clamp the resonance-tube so that it is vertical. Pour some water into the tube and lower the bottle to start the action of the siphon. Fasten the fork to some firm support, or by means of a clamp, so that its prongs are immediately over the open end of the tube. Raise the flask and allow water to run into the tube to a depth of about one centimetre. Place in the ear the end of the listening-tube and set the fork into vibration by a blow from a cork mallet. Then adjust the level of the water until the sound of the fork is reinforced by resonance from the air in the tube. The rising of the water should be stopped the instant the sound is at its loudest. Measure the distance from the level of the water surface to the upper edge of the tube. Lower the level of the water until the sound is reduced to a minimum. Determine twice more the position of the water level when maximum reinforcement takes place and measure each time the distance from the top of the water to the open end of the tube. Take the average of the three measurements as the correct length of the resonating column. Now cause the water to rise to a higher point in the tube and again set the fork into vibration. As the water slowly rises it will be noticed that the resonance gradually dies away, and then again increases gradually to a maximum. Check the rise of the water, as before, the instant the maximum sound occurs. Measure the length of the resonating column. Make altogether three determinations of the length of this column and take the average as the true length. Continue in the same way to find the length of as many more resonance columns as the remaining length of tube will give. Insert a thermometer into the upper end of the tube and take the temperature of the air. Referring to the diagram (Fig. 43):

When the prong moves from *a* to *b* how much of a vibration does it make and what part of a sound-wave is generated. What happens to this part of the wave when it reaches the lower end of the air column at the water surface?

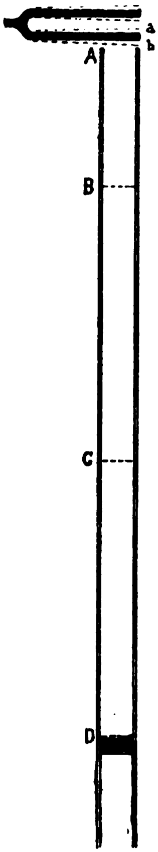

Fig. 43.

Now, if the water surface be at B, so that this part of the wave can run down the tube and return in time to combine with the part of the wave produced by the prong in moving from *b* to *a*, what will be the effect on the sound ?

How must the length of the air column AB compare with the whole length of the sound-wave produced by the fork in making one complete vibration?

When the prong moves from *b* to *a*, what part of the wave is produced? In what direction will it move? State where it will be at the time the prong returns to *a*, the length of air column remaining unchanged.

Again, if the water surface be at C, so that the prong makes 3 half-vibrations, starting from *a*, while the condensation runs down the tube and back, what must be the length of the resonance column AC expressed in quarter wave-lengths?

Will this length of air column give a reinforcement of the sound?

What fraction of a wave-length is BC?

By a similar reasoning it can be shown that the next air column, AD, that will reinforce the sound will be of such length that a condensation, or a rarefaction, will move down and back in the time that it takes the prong to make 5 half-vibrations.

Expressed in quarter-waves, what is the length of AD?

What fraction of a wave is CD?

Locate the positions of the nodes; also the antinodes.

State what is the condition of the air at each of these places when maximum resonance occurs.

Taking the results obtained by the experiment, the velocity of sound in air can now be computed. This result may be reduced to its value at 0°C., knowing that the velocity diminishes about 60 centimetres per second for each Centigrade degree lowering in temperature. Tabulate as follows:

Pitch of the fork, Temperature,

Nodes numbered from Bottom.	Lengths of Air-columns.	Differences, (½ wave-length).	Velocity of Sound.
1			
2			
3			
etc.			

Mean velocity at (state temperature), cm. per sec.
Velocity at 0° C., cm. per sec.

Since the note produced by an ordinary tuning-fork changes in intensity, thus making it difficult to determine accurately the positions of the nodes, it is better to use an electric tuning-fork or a "singing-flame." The singing flame is produced by placing a small gas flame into a glass tube held vertical in a clamp. The pitch of the tone is regulated by moving the tube up or down, or by changing its length by means of a paper cylinder sliding over the top. When the tone emitted by the air in the tube is in unison with a tuning-fork of known pitch, the fork may be laid aside and the above experiment conducted in the manner described. The tone arising from the singing flame will give a more decided resonance than could be excited by the fork and will serve for a number of experimenters working at the same time in the room.

Exercises: (1) What differences would it make in the results if a fork of different vibration frequency were used? State fully.

(2) Assuming the velocity of sound to be known, how may the vibration rate of a fork be determined by experiment?

(3) A glass jar containing water responds most loudly to a tuning-fork when the length of the column of air is 17.5 centimetres. The temperature at the time is 20°C. What is the frequency of the fork?

(4) State a reason for the fact that in a resonance tube there is always an anti-node at the open end.

(5) How long must a resonating air column be when the fork vibrates 440 times per second, and the temperature is 18°C.?

VELOCITY OF SOUND IN A SOLID

When the velocity of sound in air is known, the velocity in a solid can be found according to a method which was devised by Professor August Kundt in 1865.

A brass rod is placed in a horizontal position, and firmly clamped at its middle point, as shown in Fig. 44. To one end of the rod is fastened a disc whose diameter fits loosely into a glass tube which lies coaxial with the rod. The opposite end of the glass tube is closed by means of an easily moving piston, so that the length of the air column in the tube can be varied. When rubbed with a dry cloth, upon which a little powdered rosin has been sprinkled, the rod will be set into longitudinal vibration, and will give out a musical note of high pitch. In the vibrating rod the middle point must be a node, because, being clamped, it is a place of no motion. The two ends of the rod must be antinodes, since, being free, they are places of greatest motion. If the glass resonance tube is of suitable length, the air in it will be set in vibration

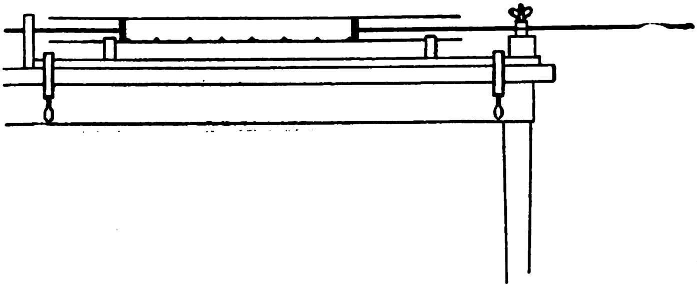

Fig. 44.

by the vibrations of the rod. As the rod alternately shortens and lengthens, the two ends moving in towards or away from the middle at the same time, the air in the tube must be alternately compressed and rarefied. The rod thus starts a series of waves down the tube exactly analogous to those which were considered in the last experiment. When these waves strike the fixed piston, they are reflected in succession, and, travelling back through the tube, interfere with the incident waves and produce *stationary* waves with their characteristic nodes and loops. In other words, there is established in the tube a series of compressions, alternating with a series of rarefactions, as represented in Fig. 45. The positions of the nodes and loops in the column of air may be made evident to the eye by placing some fine, dry powder, such as cork dust, distributed evenly along the bottom of the tube. When there are well defined nodes and loops, there are certain equidistant points in the tube where the dust is not at all disturbed. These points correspond to the nodes. Between these points of rest are the regions of greatest disturbance—the loops. At the middle point of each loop is an antinode. When the length of the air column is properly adjusted, the distribution of the dust will become well marked. Since the resonance tube is closed, both ends of the column of air are nodes.

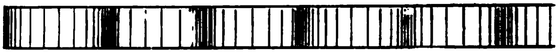

Fig. 45.

Since there is an antinode at either end of the rod and a node between, the wave-length in brass corresponding to the vibration produced in it is equal to twice the length of the rod. Also in the tube, the wave-length in air of the note given by the rod is equal to twice the distance from one dust heap to the other. The frequency of the vibrations in the column of air is the same as the frequency of the brass rod.

Let N denote the vibration frequency of the brass rod, and L its length; then, by the equation for wave motion,

$$\text{Velocity of wave in brass} = 2NL.$$

Call the distance between successive dust heaps l; then

$$\text{Velocity of wave in air} = 2Nl.$$

Hence
$$\frac{\text{Velocity in brass}}{\text{Velocity in air}} = \frac{2NL}{2Nl} = \frac{L}{l}.$$

The velocity of sound in air at 0°C. may be taken as 332 metres per second, to which 0.6 metre should be added for each degree above zero. Then, if t be the temperature of the air,

$$\text{Velocity of sound in brass} = (332 + 0.6t)\,\frac{L}{l}\ \text{metres per sec.}$$

To determine the velocity of a sound wave in a brass rod by Kundt's method.

Apparatus: (1) A glass tube about 1 metre long and 4 centimetres in diameter, resting horizontally on supports in the manner shown in Fig. 44. (2) A brass rod about 80 centimetres long and 5 millimetres in diameter, with a cork disc on one end fitting the tube loosely. (3) A piston which fits the tube tightly, yet can be inserted or withdrawn by means of a rod. (4) A clamp to hold the rod, made of two narrow blocks of wood, each having a groove across one edge for the rod to lie in. This clamp is fastened to the base of the apparatus, and can be tightened by means of screws. (5) A metre rod. (6) A joiner's clamp. (7) A small piece of rough cloth and some rosin. (8) Some cork dust.

Errors: (1) The dust heaps are not likely to be equally spaced, nor can the pitch of the sounding tube be quite uniform. (2) Owing to friction against the sides of the glass tube, the velocity of the wave is not exactly what it would be in open air. (3) Also, the vibration frequency of the brass rod is not the same as it would be if it were free.

Procedure: Thoroughly clean the glass tube, then dry it by carefully warming it over a flame or before a fire. Hold the tube inclined at a suitable angle, and by means of a knife-blade pour cork dust down one side of the tube. The dust should be made to lie as evenly as possible in a thin line through the length of the tube. Put the tube in its place, and arrange the apparatus as shown in Fig. 44. Adjust so that the brass rod shall lie coaxial with the tube and extend into it about 30 centimetres, the disc being entirely free on all sides. Set the rod in vibration by stroking it with the rosined cloth. If the dust does not gather into heaps, alter the position of the piston and again set the rod in vibration. Continue, pushing the piston in or out until the parallel heaps are most distinct. When the adjustment is properly made, the faces of the pistons will be themselves situated at nodes.

Measure the distance between two nodes as far apart as possible, and count the number of loops. Measure the distance between two other nodes at a considerable distance apart; also measure the distance between the faces of the pistons. In each case, the length measured divided by the number of loops measured will give the length of one loop. Finally, measure the length of the brass rod, and note the temperature of the air in the tube. Tabulate as follows:

Length of rod, **Temperature of air,**

No. of Loops measured.	Distance measured.	Length of one Loop.	Wave-Length.

Velocity of sound in air at observed temperature,
Mean length of the sound wave in the air in the tube,
Length of the sound wave in the brass rod,
Velocity of sound in brass,

Exercises: (1) Indicate briefly a method by which the velocity of sound in another gas than air may be found by using a Kundt's tube.

(2) Taking data furnished by the experiment, find the pitch of the note given out by the brass rod.

(3) When a Kundt's tube is filled with hydrogen it is found that the distance between the successive dust heaps is 3.8 times as great as when the tube is filled with air, the same note being sounded in each case. Compute the velocity of sound in hydrogen.

MEASUREMENT OF TEMPERATURE

THE THERMOMETER

Temperature may be measured indirectly by measuring some one of the effects produced by heat which, other things being equal, is proportional to the temperature. One of the familiar effects accompanying a change in the temperature of a body is its change of volume. In the ordinary thermometer the relative expansion of a liquid contained in a glass envelope has been adopted as the most convenient means of indicating changes in temperature. The mercury thermometer consists, in general, of a closed capillary glass tube with a bulb at one end. The bulb and a part of the tube are filled with mercury; the space above the mercury is nearly a perfect vacuum. In order to measure temperature by means of the expansion of any substance, it is necessary to fix two standard temperatures and adopt some scale by means of which the changes may be expressed. By general consent, the two conditions which determine the two standard temperatures are the temperature of melting ice and the temperature of steam rising from water boiling under a pressure of a standard atmosphere which is equal to that of a column of mercury 760 millimetres high. These two temperatures are substantially invariable. Many thermometric scales have been devised, one of which is the *Centigrade* scale proposed by Celsius and now generally used for scientific purposes. In this scale the temperature of melting ice is marked 0 and the temperature of steam 100. Were the bore of the tube perfectly uniform, it would be a simple matter to divide the distance between these two "fixed points" into 100 equal parts in order to obtain a correct scale. But, owing to the irregularities of the tube, the values of the divisions vary. In the typical form of mercury thermometer constructed for scientific purposes, the diameter of the tube is nearly uniform throughout and the divisions are made equal in length. For the sake of accuracy, however, it is necessary to determine by *calibration* the correct values of the divisions on the stem. Very briefly stated, the process consists in detaching within the tube a thread of mercury and measuring its length in different places. The various lengths obtained afford data for computing a table of corrections to be applied to the readings of the scale. It is known that the bulb of a thermometer after being strongly heated does not contract at once to its normal condition, but undergoes a gradual change for a considerable time. This change would make the fixed points incorrect even if they were originally exact. The error of the zero point may often amount to as much as 0.°5C. The instrument is, therefore, not graduated until some time after being filled with mercury.

While using the thermometer attend to the following instructions:

(1) The mercury thermometer is an exceedingly fragile instrument. The glass of the bulb is of microscopic thickness and great care will be necessary to save it from breaking.

(2) The full direct heat of a flame should never be brought near either stem or bulb.

(3) Never subject the thermometer to a sudden change of temperature. Hold it in air a short time before exposing it to a much different temperature.

(4) If any mercury has separated from the column, or if any remains in the cavity at the top of the stem, it can probably be dislodged by grasping the instrument near the middle of the stem, bulb downward, and swinging it quickly at the end of the extended arm.

(5) In testing the temperature of a liquid, it is often not best that the bulb should come in contact with the sides or bottom of the vessel. The thermometer is not intended to be used as a stirring rod.

(6) In reading the scale, the eye should be in such a position that the line joining the eye and the surface of the mercury is at right-angles to the scale. This is to avoid parallax.

(7) The thermometer must be read while it is in contact with the substance whose temperature is required. It must not be removed and then read, as is often done.

(8) When the temperature of a body is being noted, time must be given for the thermometer to reach the same temperature as the body.

To determine the freezing point.

Apparatus: (1) A Centigrade thermometer. (2) A large glass funnel. (3) An iron retort-stand. (4) A glass beaker. (5) Some clean ice shavings or snow.

Errors: (1) The ice or snow may contain impurities. (2) Water at a temperature higher than melting ice may collect near the bulb of the thermometer.

Procedure: Place the funnel on the ring of the retort stand (Fig. 46), and fill it with small pieces of clean ice. The surface of the ice should be kept somewhat above the edge of the funnel. Place the beaker underneath the funnel so as to receive the water drained off the melting ice. With the stem of the pencil make a vertical hole in the ice. The hole should be of such a depth that when the thermometer is inserted to the bottom of it, the o division is just level with the top of the ice. The thermometer is now carefully placed in the opening made. If necessary it may be fixed in this position by inserting its upper end into a cork fitted into the small ring at the top of the retort-stand. When the mercury has fallen to about 1°, note the reading every minute until it has remained stationary for five minutes, and take the final reading as the freezing point on the thermometer. Estimate carefully the tenths of a scale division. Record the result as follows:

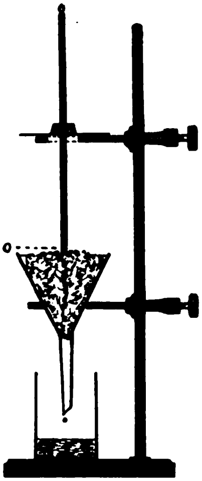

Fig. 46.

Observed freezing point, (state whether above or below the o.)

Freezing point correction, (mark it + or —, as the case may be.)

The correction at freezing point is that number which is to be *added* to the reading of the thermometer, and is marked — when the observed zero is above the o of the scale. and + when it is below.

To determine the boiling point.

Apparatus: (1) The thermometer used in the preceding experiment. (2) An apparatus consisting of a copper vessel or boiler, at the top of which is a cone-shaped sheet-copper pipe. The top of the cone is of small diameter and of such shape as to be closed by a cork stopper. A side tube leads out from the cone near the top. At the side of the boiler is attached a manometer, which consists of a U-shaped glass tube containing mercury. The whole apparatus is supported on legs in order that a Bunsen flame may be placed underneath. An improved form of the apparatus is shown in Fig. 47. The cylindrical top is fastened to the boiler by a threaded connection, fitting to an asbestos packing. The bent glass tube joined to the two side tubes by rubber tubing serves as a water gauge. The two other side tubes are to be provided with short rubber tubes and pinch-cocks for closing them. (3) A barometer. (4) A Bunsen burner.

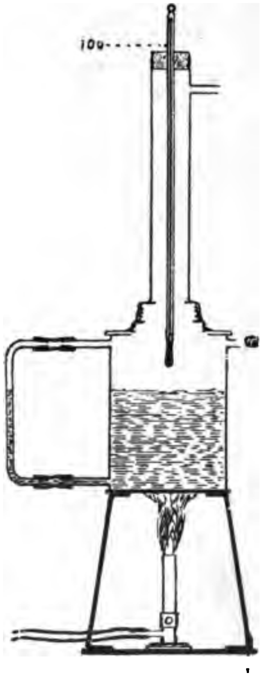

Fig. 47.

Errors: (1) The pressure around the thermometer may not be that recorded by the barometer. (2) Too large a part of the mercury column may be exposed to the air instead of steam.

Procedure: Fill the boiler with water to within about 5 centimetres of the side tube, and place beneath it the flame of the Bunsen burner. Fit the steam jacket to the top of the boiler and see that the tube for escape of the steam is open. Insert the thermometer into the cork fitted into the top of the apparatus. The bulb of the thermometer must not be too near the boiling water. The 100 mark should not be more than one or two divisions above the cork. The thermometer should fit tightly into the cork, both to prevent its falling and to prevent the escape of steam. While the thermometer is taking up its final temperature, read the barometer. Watch the thermometer for two or three minutes after the mercury has apparently stopped rising. When the top of the column has become steady, take the reading, estimating the tenths of a division. Express the barometer reading in millimetres, carrying the results to tenths. Before making the corrections for the scale of the thermometer, the temperature at which water boils under a given pressure (the temperature of steam, or boiling point) may be calculated, The boiling point is raised 0°.0375C. for each millimetre increase in the height of the barometer. Hence we have

$$T = 100 - 0.0375(760 - b),$$

in which T is the temperature of the steam and b the height of the barometer during the experiment. Tabulate as follows:

Barometer reading, cm.
Thermometer reading (scale divisions),
Temperature of steam (computed), deg.
Observed freezing point,

The remainder of the calculations may be illustrated by an example. Suppose the barometer reading is 740 millimetres; then, the temperature of steam, or boiling point expressed in degrees, will be 99.°25. Also suppose that the observed freezing point is 0.3 division below the 0 mark on the scale, and that the boiling point reads 99.6 on the scale. Then the distance between the observed freezing point and the observed boiling point, expressed in scale divisions, is 99.9. This same distance represents 99.25 degrees. Therefore, the value of each division on the scale, assuming a tube of uniform bore, is

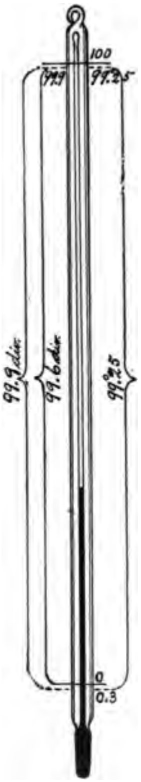

Fig. 48.

$$\frac{99.25}{99.9} \text{ of } 1°, \text{ or } 0°.993$$

From this the value of any scale reading can be expressed in degrees. For example, if the thermometer scale reads 20, the distance taken from the observed freezing point will be 20.3 divisions. To find the reading in degrees, multiply 20.3 by 0.993 This gives 20.°159 which, since only tenths of a division can be estimated, may be written 20.°16.

Exercises: (1) Using the data obtained in the experiments, compute the value in degrees of each of the following points marked on the thermometer used: 0, 50, and 100.

(2) Which experiment should be performed first; to determine the freezing point, or to determine the boiling point?

(3) State the distinction between a Centigrade degree and a scale division on a Centigrade thermometer.

(4) At what temperature will water boil at a time or place where the barometer reads 74.2 centimetres?

(5) What is the error of a thermometer which with the barometer at 76 centimetres gives the boiling point of water at 99.°4C.?

EXPANSION OF SOLIDS

COEFFICIENT OF LINEAR EXPANSION

When a solid body is heated it expands in all directions, and if there be any two points in the body exactly unit length apart, the amount by which the distance between these points increases when the body is heated 1°C. is called the *coefficient of linear expansion* of the body. For solids as a class the expansion for a rise in temperature of 1° is very nearly constant between 0° and 100° C. Suppose a rod of length l at a temperature t to be heated to a temperature t', so that its length becomes l'. The total expansion, $l' - l$, divided by the temperature change, $t' - t$, will equal the expansion of the original length for a rise in temperature of 1°. The quantity $\frac{l' - l}{t' - t}$ divided by the whole number of units of length l will give the expansion of a unit length for 1°. This is, according to definition, the coefficient of linear expansion (a). Hence we have

$$a = \frac{l' - l}{l(t' - t)}.$$

The coefficient of cubical expansion is approximately equal to three times the coefficient of linear expansion.

Various methods have been devised for the direct measurement of the small increase in length produced by expansion. The method described in the following experiment has given good results.

To determine the coefficient of linear expansion of a metal.

Apparatus: (1) A metal rod, the expansion of which is to be measured. (2) A metal tube, or steam jacket, which is closed at each end with a cork. The ends of the rod are fitted tightly into the corks and may be allowed to project about 1 millimetre beyond the ends of the jacket. At the centre of the jacket is a cork-stoppered tubulure for the insertion of a thermometer. Two small side tubes, one near each end of the jacket, are also provided. The jacket is placed in a horizontal position on a wooden frame (Fig. 49). At one end of the frame is a micrometer screw; at the other end is a short screw backstop. When the jacket is properly placed, the points of the screws lie in line with the axis of the rod and may easily be adjusted so as to bear against the ends. One of the small side tubes passes through a hole in the top of the frame, thus preventing the jacket from overturning. Binding-posts are provided so that the screws and rod may be placed in a battery circuit together with a galvanoscope. (3) A Leclanchè cell. (4) A galvanoscope. (5) A Centigrade thermometer. (6) A barometer. (7)

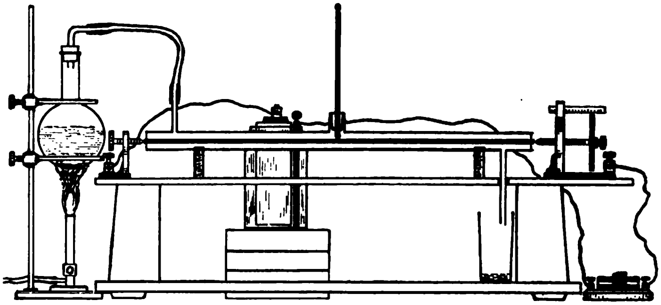

Fig. 49.

A metric scale and two rectangular blocks. (8) A flask or boiler containing water and so arranged that it can be connected by a rubber tube to the steam-jacket. (9) A Bunsen burner. (10) A beaker to collect the condensing steam issuing from the cylinder.

Errors: (1) The rod may not be of the same temperature throughout. (2) Insufficient pains may be taken in adjustng and reading the micrometer. (3) The thermometer may not agree closely with the correct scale.

Procedure: Fill the boiler about one-half full of water and place the lighted Bunsen burner beneath it. No connection should be made at this time with the steam jacket. Remove the jacket from the frame and place it horizontally alongside the metric scale. Bring the two rectangular blocks against the square cut ends of the rod and also against the edge of the scale. Read the position of the inner edge of each block; the difference between these readings will be the length of the rod. It will not be necessary to remove the rod from the jacket during the measurement. Replace the cylinder on the frame and insert the thermometer in the cork at the centre, taking care that the bulb does not touch the rod. Connect the screws at the ends of the apparatus by wires to the battery and galvanoscope. Make sure that the rod rests against the backstop screw. Slowly turn the micrometer screw till contact with the rod is indicated by the movement of the needle of the galvanoscope. Stop turning the screw at the instant the needle moves. Take the reading of the micrometer. Readjust the screw and take another reading. Record the mean of the two results. Now take the temperature observation, estimating carefully the tenths of a degree. Turn the micrometer screw back two or three revolutions to allow ample room for the expansion of the rod. Connect the jacket with the boiler so as to heat the rod to the temperature of the steam. Read the barometer and express the result in millimetres. Compute the temperature of the steam under the observed pressure. While the rod is heating, turn the micrometer screw till contact is again indicated and take the reading. Repeat the readings about ten times during the following ten minutes, each time making a readjustment of the micrometer. The mean of the last three or four readings may be recorded. Detach the heating apparatus and allow the rod to cool. Be able to answer the following questions:

What observations should be taken with the greatest care?

What is the elongation of the rod in centimetres?

Through how many degrees of temperature was the rod raised?

What is the average expansion of the rod for 1°C.?

By what part of the original length has the rod expanded for an increase in temperature of 1°C.?

Tabulate as follows:

Length of (name metal) rod, cm.
Temperature of rod when cool,
Mean of micrometer readings before expansion, cm.
Barometer reading, mm.
Temperature of rod when heated, (Computed from barometer reading.)
Mean of micrometer readings after expansion, cm.
Total expansion of rod, cm.
Temperature change,
Coefficient of linear expansion,
Coefficient given in table of reference.
Error, Percentage of error,

Exercises: (1) In taking the observations in the above experiment, which will produce the greatest effect upon the final result; (*a*) an error of one centimetre in measuring the length of the rod, (*b*) ne degree in observing the temperature change, (*c*) one-hundredth of a millimetre in measuring the expansion? Give a simple illustration.

(2) What must be the length at 50°C. of a brass standard yard measure, in order that it may be exactly correct at the freezing-point?

(3) A certain steel wire 16 feet long may be stretched 0.1 inch by a force of 2 pounds. What weight may be raised by 16 feet of the same wire while cooling from 90° to 0°C.?

(4) When the temperature rises from 0° to 30°C., how much longer will an iron bridge of 200 feet span become?

MEASUREMENT OF QUANTITY OF HEAT

WATER EQUIVALENT

Heat is a physical quantity in the same sense that force is a quantity. If two equal masses of a substance, both at the same temperature, be raised to the same higher temperature it is evident that the same quantity of heat has been given to each. If the two masses be put together they will require twice as much heat to raise their temperature through any number of degrees as is required to raise either mass separately through the same range of temperature. For example, to raise 20 grammes of water from 0°C. to 5°C. will require twice as much heat as to raise 10 grammes of water from 0°C. to 5°C. In practice a quantity of heat is expressed in terms of the change it will produce in the temperature of a definite mass of water. The unit of heat quantity generally used is that amount of heat which is required to raise the temperature of one gramme of water 1°C. This unit is called a *caloric*. Careful experiments show that the quantity of heat required to raise the temperature of a given mass of water 1°C. is very nearly the same at different temperatures. Furthermore, the amount of heat given out by a mass of water in cooling a certain number of degrees is equal to the amount of heat gained by the same mass of water in heating through the same range of temperature.

An apparatus used for measuring quantities of heat is called a *calorimeter*. It is often required to determine the mass of water which would absorb the same amount of heat when warmed through a certain number of degrees as is absorbed by a calorimeter when heated through the same range of temperature. This quantity is called the *water equivalent* of the calorimeter. The water equivalent of any body may be defined as the number of grammes of water which will be raised 1°C. by the heat required to raise the temperature of the body 1°C. In experiments requiring great accuracy it is necessary to determine not only the water equivalent of the calorimeter, but also the water equivalents of thermometers, stirring rods, etc.

To determine the water equivalent of a calorimeter.

Apparatus: (1) A calorimeter (Fig. 50). This consists of a small vessel made usually of thin sheet copper polished on its outer surface. The vessel is supported on cork and placed inside a larger vessel which protects it against irregular changes of temperature due to air currents and to some extent from loss of heat due to radiation and conduction. (2) A Centigrade thermometer. (3) A platform balance and a set of metric weights. (4) A flask for heating water. (5) A retort stand.

Errors: (1) The warm water is cooled by contact with the air while it is being poured into the calorimeter. (2) The calorimeter loses part of its heat by radiation and conduction to the air.

Procedure: Carefully dry the calorimeter and find its weight to tenths of a gramme. Then place a thermometer in the calorimeter to register its temperature. Heat a quantity of water to a temperature of about 30° C. After five minutes note the temperature of the calorimeter. Place the thermometer in the water and use it

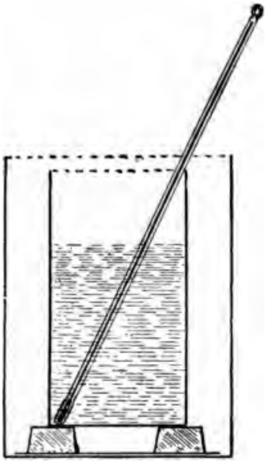

Fig. 50.

carefully as a stirrer. When the water has reached the required temperature, remove the flame and stir the water well. When a steady temperature is reached, pour the water quickly, but carefully, into the calorimeter, until the latter is about one-half full. During this process keep the thermometer bulb in the water and insert it at once into the calorimeter. Stir well for a time until the first fall in temperature appears to cease and note the thermometer reading. Remove the thermometer and weigh the calorimeter and the contained water to find the mass of water poured in. Take the results obtained and prepare to answer the following:

Through how many degrees has the temperature of the water been lowered?

How much heat has been given out by one gramme of the water in cooling 1°C.?

How much heat has been given out by the whole mass of water in cooling 1°C.?

How much heat has been given out by the water in cooling the whole number of degrees?

How much heat has been required to raise the temperature of the calorimeter?

How many degrees has this temperature been raised?

How much heat was needed to raise the temperature of the calorimeter 1°C.?

How much water would be raised 1°C. by this quantity of heat?

What is the water equivalent of the calorimeter?

Write out the results thus:

Mass of calorimeter,
Temperature of calorimeter,
Temperature of water before pouring in,
Temperature of water after pouring in,
Mass of water poured in,
Water equivalent of the calorimeter (*w*),

Consider this value of the water equivalent of the calorimeter as so many grammes of water in addition to the mass of water in the calorimeter. Make an equation, using data derived from the experiment; on one side write the expression for the quantity of heat given out by the water in cooling, on the other, the quantity of heat received by the calorimeter. For the water equivalent in the equation write *w*. Divide the value of *w* by the weight of the calorimeter. This will give the amount of heat required to raise 1 gramme of the substance of which the calorimeter is made 1°C. Write this result in your note book thus:

Fraction of calorie required to raise 1 gramme of (name metal of calorimeter) 1°C. =

Exercises: (1) What is the reason for not having the water at a very high temperature when poured into the calorimeter?

(2) Account for the gradual cooling that takes place after the first rapid fall in temperature.

(3) Through how many degrees will 480 grammes of water be raised by 1680 calories?

(4) A block of hot iron is dropped into a vessel containing 230 grammes of water at 15°C., and raises the temperature of the water 25°.6C. How much heat was given out by the block?

(5) A calorimeter contains 20 grammes of water at 10°; 40 grammes of water at 40° are poured in, and the temperature after stirring is found to be 28°. What is the water equivalent of the calorimeter?

SPECIFIC HEAT

To raise the temperature of 1 gramme of water 1°C. requires a unit quantity of heat called the calorie. It is shown by experiment that this quantity of heat will raise the temperature of 1 gramme of almost any other substance more than 1°C. Thus, a unit quantity of heat will raise a gramme of lead about 30°C., a gramme of tin about 20°C., a gramme of copper 10°C. Conversely, it will be seen that different substances require very different quantities of heat to raise the temperature of a given mass through a given number of degrees. Also, equal masses of different substances in cooling the same number of degrees give out different quantities of heat. This may be shown as follows: Several balls of different metals and of about the same mass are placed in boiling water. After thorough heating the balls are removed simultaneously from the water and placed on a cake of beeswax. The balls will melt the wax, but the amount melted by each will be different. Suppose the metals taken are lead, tin, and copper. After a time, when all melting has ceased, it will be found that the copper ball has sunk deepest in the wax, while the tin ball has penetrated a shorter distance and the lead has melted less wax than either of the others. It appears then that equal masses of the three different substances cooling through the same number of degrees give out different amounts of heat. This fact is expressed by saying that these substances differ in *specific heat.* In general, the specific heat of any substance is the ratio of the quantity of heat required to raise the temperature of a given mass of that substance 1° C. to the quantity of heat required to raise the temperature of an equal mass of water 1°C. Or, since the quantity of heat required to raise 1 gramme of water 1° C. is the calorie, the specific heat of any substance is simply the number of calories necessary to raise the temperature of 1 gramme of that substance through 1°C.

To determine the specific heat of a solid.

Apparatus: (1) A calorimeter. (2) A Centigrade thermometer. (3) A platform balance and set of metric weights. (4) Small pieces of the solid whose specific heat is to be found. (5) A large test-tube fitted into a flat cork so as to be placed vertically in a water bath (Fig. 51). Near the test-tube is a perforation in the cork for the insertion of the thermometer. (6) A water bath, an iron tripod, a Bunsen burner.

Errors: (1) The metal cools very rapidly when it is transferred from the bath of boiling water to the calorimeter. (2) Heat is lost by radiation and conduction from the calorimeter to surrounding bodies, assuming the calorimeter at a higher temperature than the room, or gained by the calorimeter from those bodies if the calorimeter be cooler than they are.

Procedure: Place the water bath filled about two-

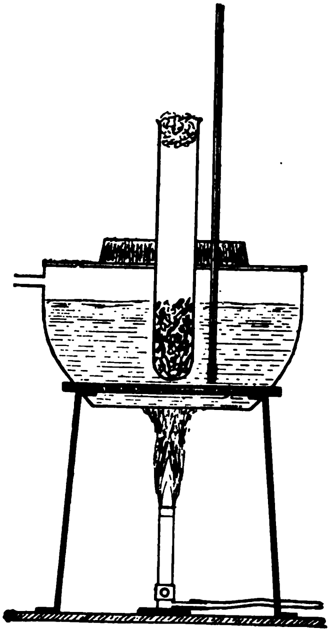

Fig. 51.

thirds full of water over a flame to heat. Take about 100 grammes of the substance, broken into small pieces, and weigh it to 0.1 gramme. Put the substance into the test-tube, avoiding the possible danger of breaking the glass. Close the mouth of the tube with a plug of cotton. Place the cork over the bath and insert the tube deep enough into the boiling water to surround all of the substance. Diminish the flame of the burner so as to keep the water slowly boiling. Weigh the empty and dried calorimeter. Fill it about three-fourths full of water and again weigh to find the mass of water. Place the calorimeter upon the cork inside of the larger vessel. When the water in the bath has boiled for about five minutes, insert the thermometer in an opening near the test-tube and take the temperature of the boiling water, carefully estimating the tenths of a degree. Remove the thermometer from the water bath, cool it in some cold water in a beaker and wipe it dry. Then place it in the water in the calorimeter, and note the temperature of the water. Leaving the thermometer in the calorimeter, quickly take the test-tube from the bath of boiling water and empty the substance into the calorimeter as rapidly as possible. Stir the water carefully and thoroughly by means of the thermometer, and note the temperature when the mercury column has risen to its greatest height and is just beginning to recede. By means of the data obtained, prepare to answer the following questions:

What is the rise in the temperature of the water?

How much heat has the water received?

What is the water equivalent of the calorimeter?

How much heat has the calorimeter received?

What is the amount of heat received by the water and the calorimeter together?

Where did this heat come from?

How much heat has been given out by the solid?

How many degrees has the temperature of the solid fallen?

How much heat was given out by the solid in cooling 1°C.?

How much heat was given out by 1 gramme of the solid in cooling 1°C.?

What then is the specific heat of the solid?

The results may be tabulated as follows:

Mass of (name substance),
Mass of calorimeter,
Water equivalent of calorimeter,
Mass of water,
Temperature of solid,
Temperature of water,
Temperature of mixture,
Specific heat of (name substance),
Specific heat given in table,
Percentage of error,

Make an equation, employing the results obtained; on one side write the expression for the quantity of heat given out by the solid in cooling, on the other, the quantity of heat received by the water plus the quantity of heat received by the calorimeter. For the specific heat of the solid write S, and show the equation solved for the value of S.

Exercises: (1) Compare the amount of heat required to warm one kilogramme of the metal from 10°C. to 90°C. with the amount of heat required to warm the same mass of water the same number of degrees.

(2) Compute the water equivalent of the calorimeter, assuming that it is composed entirely of copper.

(3) What is the specific heat of a substance if the same amount of heat is needed to raise the temperature of 30 grammes of it from 80°C. to 90°C. as is needed to raise 10 grammes of water from 90°C. to 100°C.?

(5) 354 grammes of water at 25°C. are contained in a calorimeter whose water equivalent is 6. 5 grammes of copper at 200°C. are dropped into the water. Find the resulting temperature.

LATENT HEAT OF FUSION

When heat is applied to a solid, its temperature rises till it reaches the point where it begins to pass into the liquid form. Any further supply of heat produces no rise in temperature while the melting is in progress; the heat is used up in melting the solid. The heat absorbed by the solid is energy converted into the potential form in the work of giving mobility to the molecules, and is said to become "latent" or hidden. As soon as the solid is melted, a continued application of heat causes the temperature of the liquid to rise. Conversely, when the temperature falls, a stationary point is again reached where solidification sets in, and the heat rendered latent on melting is set free again. Under the same conditions of pressure, the two stationary temperatures, that of melting and that of solidification, coincide. The quantity of heat required to convert 1 gramme of a substance from the solid to the liquid state without change of temperature is called the *latent heat of fusion* of the substance.

To determine the latent heat of fusion of ice.

Apparatus: (1) A calorimeter. (2) A platform balance and metric weights. (3) A Centigrade thermometer. (4) Some ice. (5) A beaker in which to heat water. (6) A tripod and a Bunsen burner.

Errors: (1) Some water may be carried into the calorimeter with the ice. (2) Water may be lost by splashing while stirring or putting in the ice. (3) If the initial temperature of the calorimeter be many degrees above or below the temperature of the surrounding air, heat may be lost or gained by radiation or conduction.

Procedure: Dry and weigh the calorimeter. Heat a quantity of water to about 35° in the beaker. While the water is heating prepare several pieces of clean ice and place them upon a cloth or piece of blotting-paper to absorb the water formed in melting. Pour the water into the calorimeter to within about 3 centimetres of the top, and weigh to find the mass of water taken. Stir the water with the thermometer; and when the temperature is about 30°, begin to add dry pieces of ice. Note the temperature of the water at the very instant before the first piece of ice is put in. Put the ice in slowly, in the meantime stirring with the thermometer, but never let the ice melt before more is put in. Add ice until the temperature when all ice is melted falls to about 10°. Take the reading of the thermometer and weigh the calorimeter and contents again to find the mass of ice added. By means of the data obtained prepare to answer the following questions:

How much heat has the warm water given out in cooling?

How much heat has the calorimeter given out in cooling?

Part of this total heat has caused the ice to melt, and the rest has raised the temperature of the melted ice.

What is the rise in the temperature of the melted ice?

How much of the heat given out by the water and the calorimeter has gone to raise the temperature of the melted ice?

How much heat has been used in causing the ice to melt without change of temperature?

How much ice was melted?

How much heat was needed to melt 1 gramme of ice without changing its temperature?

What is the latent heat of fusion of ice?

The results may be tabulated as follows:

Mass of calorimeter,
Water equivalent of calorimeter,
Mass of water taken,
Mass of ice added,
Initial temperature of water and calorimeter,
Final temperature after mixing,
Latent heat of fusion of ice
Percentage of error,

Make an equation, employing the results obtained; on one side write the expression for the quantity of heat necessary to melt the ice plus the quantity of heat necessary to raise the ice-water to the temperature of the mixture, on the other, the quantity of heat given off by the water in cooling to the temperature of the mixture plus the quantity of heat given out by the calorimeter in cooling to the same temperature. For latent heat of fusion write L.

Exercises: (1) If 1 kilogramme of ice at 0°C. is mixed with 1 kilogramme of water at 80°C., what will be the result?

(2) If 200 grammes of water at 35°C. are poured into 70 grammes of ice at 0°, what will be the temperature of the water resulting?

(3) How much ice must be added to 5 kilogrammes of water at 60°C. in order that the temperature when all ice is melted may be 50°C.?

(4) A lump of iron of mass 750 grammes at a temperature of 400°C is placed on a block of ice. 420 grammes of water are produced; what is the specific heat of iron?

LATENT HEAT OF VAPORIZATION

The passage of a liquid into the form of a gas or a vapor is called *vaporization.* Vapor may be formed solely at the free surface of the liquid, or it may be formed in bubbles throughout the entire mass of the liquid. Whenever a liquid is heated in an open vessel, more rapid vaporization goes on from its surface as the temperature rises. Vapor is soon liberated in bubbles from the walls of the vessel beneath the surface. When the liquid mass becomes so hot that the bubbles reach the surface without condensing, ebullition or boiling begins. The space above the liquid is filled with vapor, and the pressure of the vapor is equal to the pressure of the atmosphere which it supports. If heat be supplied at a more rapid rate, the temperature of the liquid does not rise higher, but the boiling becomes more violent. The temperature of boiling remains constant under constant pressure until all the liquid has been vaporized. That portion of the heat energy which is imparted to a liquid to vaporize it is expended in overcoming the cohesion of the molecules and is stored up in the vapor as potential energy. When, on the other hand, a vapor is condensed, the stored energy becomes available again. The heat consumed during the vaporization is said to be *latent.* The *latent heat of vaporization* is the quantity of heat required to make one gramme of a liquid mass pass into the state of a vapor at a definite temperature.

To determine the latent heat of vaporization of water at the boiling point.

Apparatus : (1) A calorimeter resting on cork inside of a larger vessel. (2) A platform balance and metric weights. (3) A 500 cc. flask provided with a stopper through which passes a glass delivery tube bent twice at right angles (Fig. 52). The end of this tube passes through a stopper in a side-neck test-tube which is to serve as a trap to collect any hot water formed from

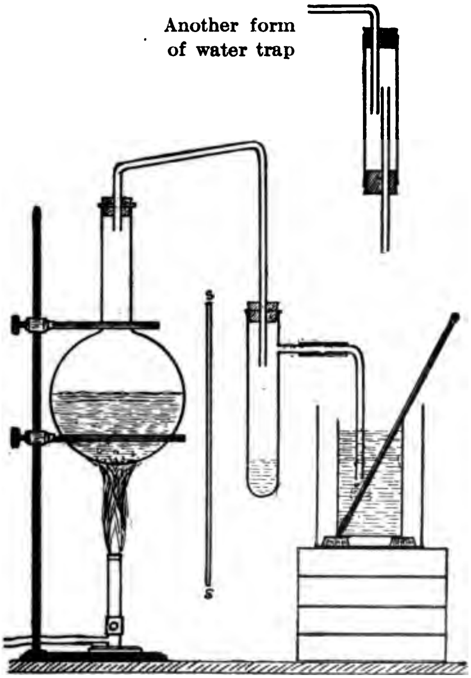

Fig. 52.

steam condensing in the delivery tube. Joined to the side tube near the top of the water trap is a short elbow tube which is to deliver the steam into the calorimeter. (4) A barometer. (5) A Centigrade thermometer. (6) A retort stand and Bunsen burner. (7) A sheet of paste board to serve as a shield between the flame and the calorimeter.

Errors: (1) There is always some loss of heat on account of radiation and conduction. (2) The steam may not be dry as it enters the water.

Procedure: Fill the flask about one-half full of water, attach the delivery tube and water trap, and adjust the whole on the retort stand with the Bunsen flame under the flask. Weigh the calorimeter, then fill it to within 2 centimetres of the top with water cooled to about 15°C. below the temperature of the room. Weigh again to find the mass of the water taken. As soon as there is a strong flow of steam through the trap, stir the cold water thoroughly, take its temperature, then quickly place the calorimeter in position, supported on wooden blocks if necessary, so that the tube leading from the trap reaches about 2 centimetres below the surface of the water. Keep the screen between the flame and the calorimeter. Stir the water with the thermometer as the temperature rises. When the temperature has been raised to about 35°C., lift the whole apparatus quickly away from the calorimeter, stir the water quickly and thoroughly, and take the temperature. Weigh the calorimeter and contents to find the mass of the steam condensed. This weighing is very important and should be performed with great care. The temperature of the steam may be computed by the formula

$$T = 100 + 0.0375\ (b - 760).$$

in which *b* represents the reading of the barometer expressed in millimetres. From the data obtained, compute the quantity of heat which has been given to the cold water and the calorimeter. Notice that this quantity of heat is composed of two parts; one given out by the steam in condensing to water, the other given out by the condensed steam (water at its boiling point) in cooling to the temperature of the mixture.

What is the quantity of heat given out by the condensed steam in cooling to the temperature of the mixture?

What is the quantity of heat which has been given out by the steam in condensing into water without change of temperature?

How much heat has been given out by one gramme of steam in condensing without change of temperature?

What is the latent heat of vaporization of water?

The results may be tabulated as follows:

Mass of the calorimeter,
Water equivalent of calorimeter,
Mass of cold water taken,
Mass of steam added,
Initial temperature of water,
Initial temperature of steam,
Temperature of mixture,
Latent heat of vaporization of water,
Percentage of error,

Make an equation, based on the above results; on one side write the expression for the quantity of heat received by the cold water plus the quantity of heat received by the calorimeter, on the other, the quantity of heat given up by the steam in changing into water plus the quantity of heat given up by the condensed steam in cooling to the temperature of the mixture.

Exercises: (1) What is the reason for having the final temperature of the water as much above the temperature of the room as the initial temperature of the water is below it?

(2) How much heat is required to convert 50 grammes of water at 12°C. into steam at 100°C.?

(3) How many pounds of steam at 100°C. will just melt 50 pounds of ice at 0°C.?

(4) How much heat will be set free by condensing 200 grammes of steam at 100°C., cooling the water to 0°C., freezing it, and finally chilling it to 8° C.? (Sp. heat of ice is 0.505.)

(5) Why is steam such an effective agent in heating buildings?

INTENSITY OF ILLUMINATION

Consider a surface on which light is incident; the illumination per unit area of that surface will depend upon two conditions; the illuminating power, or energy, of the source and the position of the surface with respect to the source. If the source of light waves is a radiant point, the waves spread out in all directions, and if the medium is "isotropic," *i. e.* has the same properties in all directions, the waves all travel with the same velocity—regardless of wave-length—and the "wave-front" is spherical. Let two spheres be drawn around the source with radii a and b; then the energy passing through each sphere is the same, but the amount per unit area of surface is different. The area of the spherical surface of radius a is $4\pi a^2$; that of the surface of radius b is $4\pi b^2$. Hence, if E is the total energy radiated from the source, the energy per unit area at the surface of radius a is $\frac{E}{4\pi a^2}$, and that at the surface of radius b is $\frac{E}{4\pi b^2}$. The amount of energy radiated by the "point-source" through one square centimetre in one second is called the *intensity of the illumination.* Then if I be the intensity at a, and i the intensity at b,

$$I : i = \frac{E}{4\pi a^2} : \frac{E}{4\pi b^2},$$

or

$$I : i = \frac{1}{a^2} : \frac{1}{b^2}.$$

That is to say, *the intensity of illumination at a surface due to a given source of light varies inversely as the* *square of the distance between the surface and the source.* This is the law of inverse squares for light.

To verify the law of inverse squares.

Apparatus: (1) An optical bench. This is an arrangement by means of which blocks carrying various pieces of optical apparatus, such as screens, photometer discs, lenses or mirrors can be made to slide backwards and forwards in a straight line. The distances between the various pieces of apparatus are determined by means of a scale attached to the bar on which the blocks slide. Each block carries a pointer which moves along the edge of the scale. If then the apparatus supported by each block be placed vertically above the pointer, the required position of a given piece can be directly determined. (2) A candle. (3) Three cardboard screens, designated by letters A, B, C, each fastened to a block in a vertical position. Screen A has a hole about 0.5 centimetre in diameter sharply cut through its centre. Screens B and C are covered with cross-section paper. A half inch square is cut out of the centre of B by following the ruled lines with a sharp blade. (See Fig. 53)

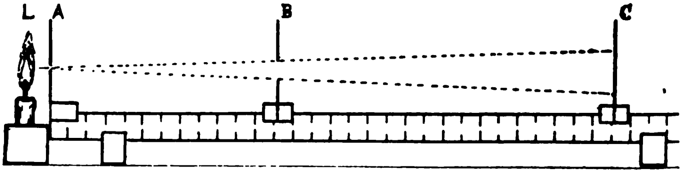

Fig. 53.

Errors: The edges of the area of light thrown on C should be kept as distinct as possible. To aid in doing this, the brightest part of the flame, the hole in A, and the middle of the opening in B must be at equal heights.

Procedure: Perform the experiment in a darkened room. Place the lighted candle on a block at the end of the scale, and at a short distance from it place screen A. Make an observation with screen B at 10 centimetres from the light and C at twice that distance. Find the number of squares included in the area of light on C. Repeat the observation with C at three, and again at four times the distance of B from the light. Make another series of observations, beginning with B at a greater distance from the light, say 15 centimetres, and place C at positions successively 30, 45 and 60 centimetres from the light. Now, with screens A, B, and C at any relative positions given by the experiment, recall the meaning of intensity of illumination and consider the following:

Compare the amount of light falling on C with the amount passing through the opening in B.

Compare the area of illumination on C with the area of the opening in B.

Compare the intensity of the illumination at C with the intensity of the illumination at B.

Express the ratio between these intensities of illumination in terms of the distances of the screens from the light.

The results may be tabulated as follows:

Distances.		Areas.		Ratio of Intensities.	Ratio of Distances. (squared)	Percentage Differences.
B	C	B	C	B : C		

Exercises: (1) In what other groups of phenomena is the law of inverse squares known to apply?

(2) Compare the intensity of the light received by a card

at a disance of 25 centimetres from the source of light and the intensity on the same card at a distance of 75 centimetres.

(3) The intensity of illumination at a unit distance from a standard candle is 1. What will express the intensity of its illumination at distances of 3, 5, and D from the light?

(4) Two lights, *a* and *b*, illuminate opposite sides of a screen with equal intensity; one is one metre distant, the other is five metres distant from the screen, how much brighter is one light than the other?

PHOTOMETRY

The general method of photometry is based on the law of inverse squares. Let opposite sides of a screen be illuminated, the one by a given source of light, the other by a source which is to be compared with it. When the distance of either light is adjusted until both sides of the screen appear equally illuminated, the intensity of illumination due to each light is the same. If the distances of the two lights from the screen are *a* and *b*, and if the illuminating powers are P and S, the intensities of illumination are respectively

$$\frac{P}{a^2} \text{ and } \frac{S}{b^2}.$$

Since these two intensities are equal,

$$\frac{P}{a^2} = \frac{S}{b^2}, \text{ or } \frac{P}{S} = \frac{a^2}{b^2}.$$

Hence the illuminating powers of two lights are proportional to the squares of their distances from a surface equally illuminated by each.

Several methods of testing the equality of illumination at a surface have been devised. In the form of apparatus known as Bunsen's photometer, the test is made by the aid of a paper disc with a translucent spot at its centre. If such a disc be viewed by reflected light, the central spot will appear darker than the rest, since a portion of the incident light is transmitted. If the disc be viewed by transmitted light, the spot will appear lighter than the surrounding area, since more light comes through it. If, therefore, both sides of the disc be equally illuminated, the loss of light due to transmission from one side is counterbalanced by the amount of light transmitted from the other side. Under these conditions, the intensities of the two sources of light are directly as the squares of their distances from the disc.

To compare the intensities of illumination of two lights by means of a Bunsen photometer.

Apparatus: (1) An optical bench. (2) A Bunsen photometer. Both sides of the disc may be viewed by the aid of two plane mirrors, one on either side, so inclined that they reflect to the eyes the two images of the oiled spot simultaneously. (3) A block holding a small candle. (4) A fish-tail gas burner. (A kerosene lamp may be used as well.) (5) Two tin screens, with equal rectangular openings. (See Fig. 54)

Errors: (1) All extraneous light should as far as possible be excluded from the photometer. (2) The illuminating power of each light should be maintained uniform. Wicks must be kept well trimmed.

Procedure: Perform the experiment in a darkened room. Mount the candle and the gas flame upon the optical bench at a distance of about 90 centimetres apart. At about the middle of the bench place the photometer, with the plane of its disc perpendicular to the line joining the lights. See that the lights are both in the same horizontal line with the centre of the disc. Turn the flame of the gas burner flatwise toward the photometer. Immediately in front of each light fasten a tin screen at

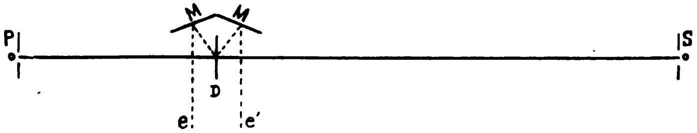

Fig. 54.

such a height that the opening will expose the brightest part of the flame to the photometer. Find, by moving the photometer back and forth, a position such that the illumination is apparently equal on both sides of the disc. Measure the distances from the disc to the lights. Displace the photometer to the right, and repeat the adjustment and the observations. Displace to the left, and repeat a second time. Represent the means of the distances by *a* and *b*. Take another set of observations in the same manner with the two lights at a different distance apart. Represent the means of the distances found by A and B. Then calculate the relative illuminating powers of the two lights, P and S, by the equations

$$\frac{P}{S} = \frac{a^2}{b^2} = \frac{A^2}{B^2}.$$

The tabulation may be arranged as follows:

	a	b		A	B
Mean,			Mean,		

$\frac{a^2}{b^2} =$ $\frac{A^2}{B^2} =$

Mean =

Exercises: (1) Distinguish between illuminating power and intensity of illumination.

(2) Two equal sources of light are placed on opposite sides of a screen, one being 20 centimetres from it, and the other 30 centimetres. Compare the intensity of illumination on the two sides of the screen.

(3) How many candles at a distance of 40 centimetres will produce the same illumination as one candle at 25 centimetres?

(4) Compare the illuminations produced by a lamp of 10 candle power at 15 centimetres and a lamp of 25-candle-power at 10 centimetres.

REFLECTION OF LIGHT AT A PLANE SURFACE

When waves of any kind passing through a medium meet the bounding surface of a second medium, they are always in part reflected according to certain laws. If the surface is smooth as in a perfect mirror, reflection will be regular. Consider a simple case, that of spherical waves from a point-source falling upon a plane surface AB (Fig. 55). Let the spherical wave-front proceed from the point P, meeting the surface at C, where PC is perpendicular to the plane surface. If there had been no reflecting surface, the wave-front would have reached the position MON in a certain definite time; but owing to reflection the wave-front becomes MQN. This new wave-front will proceed back from the surface exactly as if it came from the point R below the surface. The portion of the waves from P which meet

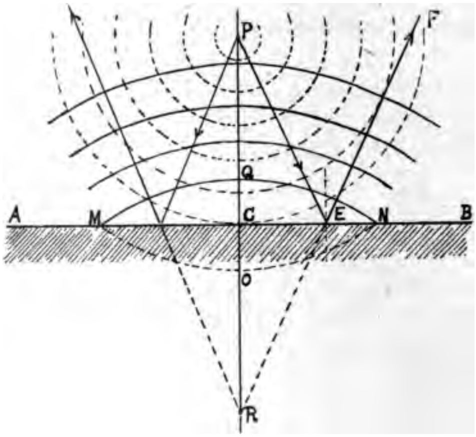

Fig. 55.

the surface at E may be called a "ray" incident on a tiny plane surface. By reflection the path of the ray PE is changed into EF, which is a prolongation of RE. Since PC = CR, it follows from geometry that the angles PEC and FEB are equal. If a *normal* be drawn at E, it can be further demonstrated that the angle between the incident ray and the normal, called the *angle of incidence,* is equal to the angle between the reflected ray and the normal, called the *angle of reflection.*

The old conception of light "traveling in rays" will be retained in these experiments only so far as it is an aid in giving descriptions, and the word "ray" will be understood to mean nothing more than the line along which a bit of the wave-front travels.

To verify the law of reflection from a plane surface.

Apparatus: (1) A plane glass mirror about 15 centimetres long and 5 centimetres wide. (2) A small rectagular block of wood. (3) A sheet of paper about 50 centimetres square. (4) A metric scale. (5) Some pins.

Errors: (1) The mirror may not be perfectly plane. (2) If a thick plate mirror be used, the refraction through the glass may produce some error. (3) Confusion may arise due to more than one reflecting surface. A reflection from the bare surface of the glass may not always be distinguished from a reflection from the silvered surface.

Procedure: Fasten the sheet of paper upon the table and along one edge of it draw a straight line. Attach the mirror to the rectangular block by means of a rubber band, and place the block so that the reflecting surface of the mirror shall be perpendicular to the paper and lie directly over and parallel to the line. Also arrange so that the middle of the mirror is near the middle point of the line. Set two pins vertically into the paper at two points about 10 centimetres apart and at such positions that the straight line passing through them will fall obliquely upon the mirror. On looking into the mirror, the reflections of the two pins will be seen. Move the head about until, when looking with one eye on a level near the paper, the reflections of the two pins appear to be in the same straight line. Keeping the eye in this position, set two other pins into the paper so that they may also appear to be in the same straight line as the two reflections. Remove the mirror and draw two straight lines, one passing through each pair of pins. These lines should intersect at a point on the line which marks the reflecting surface. At this point draw a perpendicular to the line. The angle between this perpendicular and the line drawn through the first pair of pins is the *angle of incidence,* and the angle

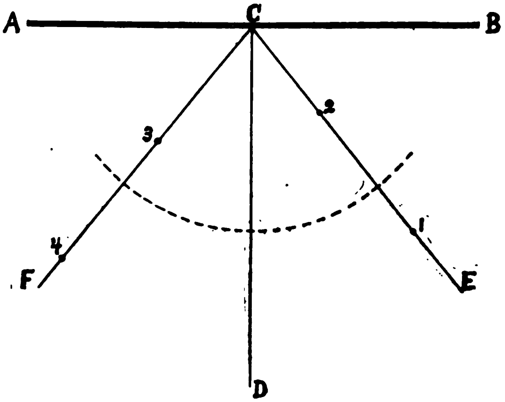

Fig. 56.

between the perpendicular and the line through the second pair of pins is the *angle of reflection.* To find whether these angles are equal, draw around the point of incidence a circle of about 15 centimetres radius, and compare the lengths of the intersected arcs (Fig. 56). Or, on CE and CF make CM equal to CN and draw MN cutting CD in R (Fig. 57). Then if it be found by measurement that MR is equal to NR, the triangles CMR and CNR are equal in all respects and the angle MCR is equal to the angle NCR.

Repeat the experiment twice, taking different positions for the pins each time.

To show that the incident ray, the reflected ray, and the normal all lie in the same plane, arrange the pins on CE so that their heads may be at the same height above the surface of the paper. Then adjust the heights of the pins on CF so that the tops of the four pins appear in a line. Measure from the paper to the top of each pin, and if it be found that all four pins are at the same height, it follows that both rays are parallel

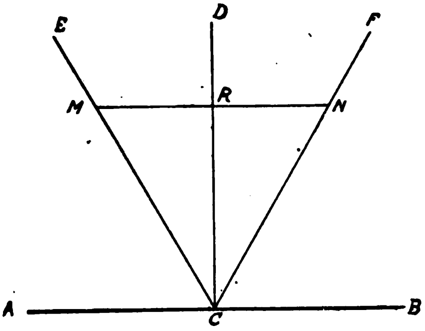

Fig. 57.

to the surface of the paper. According to the conditions of the experiment, the normal at C must also lie in a plane parallel to the surface of the paper. Hence the incident ray, the reflected ray, and the normal all lie in one plane.

Exercises: (1) A ray of light is reflected successively from two mirrors inclined at right angles to each other. Prove that the ray after a second reflection is parallel to its original direction.

(2) What is the angle between a plane mirror and an incident ray when the angle between the incident and reflected rays is 50°?

(3) When a plane mirror is turned about an axis in its own plane, show by a diagram that the change in angle between the incident and reflected ray is double the angle through which the mirror turns.

(4) If a luminous point P is placed at a very great distance from a plane mirror, the rays falling upon the mirror will be practically parallel. Show that in this case the reflected rays will also be parallel to each other.

IMAGES FORMED BY REFLECTION AT A PLANE SURFACE

Light-waves diverging from a point may be caused by reflection either to converge toward or to appear to diverge from a second point. This second point is in either case called the *image* of the first point. Images are either *real* or *virtual.* If the waves proceeding from a point are made by reflection to converge toward a second point, the second point is said to be a real image of the first point. If the waves after reflection only appear to diverge from a second point, the second point is said to be a virtual image of the first point. In other words: when rays after reflection actually pass through the image, the image is real; when rays after reflection do not actually pass through the image, but would if produced backward pass through it, the image is virtual. To an observer looking into a plane mirror the image appears in the direction of the ray from the face of the mirror to the eye; and waves diverging from an image produce vision of that image in the same way as if it were actually a source of light. The image of an object consists of the images of the various points which form the object.

To find the position of the image of a point seen by reflection at a plane surface.

Apparatus: Same as in the preceding experiment.

Errors: As in the preceding experiment.

Procedure: (I) Fasten the sheet of paper upon the table and across the middle of the sheet draw a straight line AB (Fig. 58). Place a mirror as before upon the line, and at a point about 20 centimetres from the mirror place a pin P.

Sight along the surface of the paper and place two pins in such positions that the points of the pins and the image of P are in the same straight line. Mark the positions of the two pins 1 and 2. Leaving the mirror and P undisturbed, repeat the experiment with the eye in a different position. Mark the positions of the pins 3 and 4. Remove the mirror and draw the lines joining the points 1 and 2, 3 and 4, etc., extending them back of AB until they meet at a common point. Mark the point of intersection R. The rays of light diverging from P appear after reflection to diverge from R. Join P and R by a straight line. Consider the results obtained as follows:

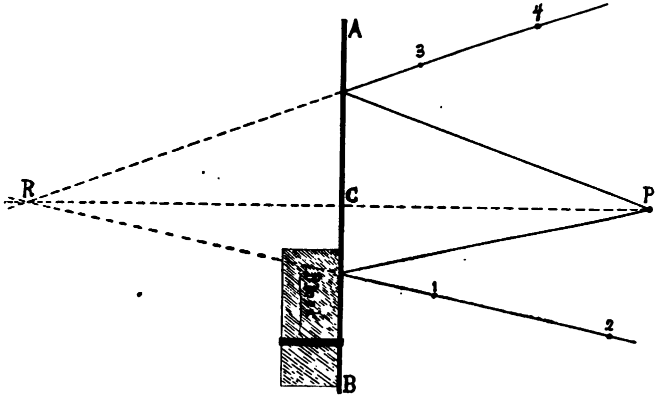

Fig. 58.

What kind of an image is formed? Why?

How does the line AB divide the line PR?

What angle do the lines AB and PR make with each other?

Define in exact terms the position of the image with respect to the position of the point.

Repeat the experiment twice, placing the pin P at a different position in front of the mirror each time.

Two objects at the same distance from an observer do not appear to move relatively to each other when the eye is moved to one side or the other, but on looking at two objects not at the same distance and moving the eye to either side, the nearer object will appear to move in the opposite direction. Thus on looking at two vertical pins, one behind the other, when the eye moves to the left the nearer pin appears to move to the right, and when the eye moves to the right the pin moves to the left. This shifting of one object past the other is called *parallax,* and suggests a direct method for locating the position of an image behind a mirror.

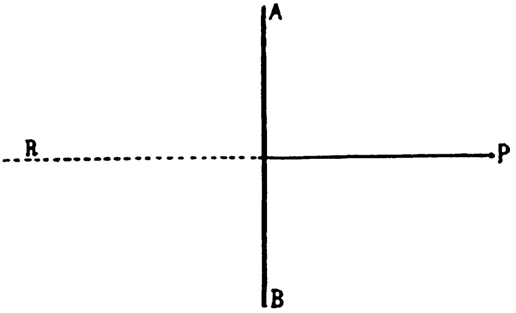

Fig. 59.

(II) Prepare another sheet of paper and place the mirror upon it as before. Take two pins each of which will reach somewhat above the top of the mirror. Set one pin into the paper about 15 centimetres in front of the mirror at P. Then looking toward the mirror so that P and its image appear immediately one behind the other, . place the other pin somewhat behind the mirror, as at R, so that the top of the pin appears continuous with the image of P seen in the mirror. R may be in front of or behind the image. Now move the eye to the left. If the pin R appears to the right of the image, it must be too near the mirror; if to the left, it must be beyond the image. By a succession of trials, a position of the pin R will be found such that, viewed from any direction towards the mirror, it will appear continuous with the image of P. R is now exactly at the position in which the image of P is formed. Join the points P and R by means of a line cutting AB at C.

Measure the distances PC and RC. Are they equal?

What angle do AB and PR make with each other?

Are the results obtained by the first method verified?

Exercises: (1) A plane mirror is inclined to the floor of a room at an angle of 45°. What will be the position of the image of a man standing before it?

(2) A man 6 feet high sees his image in a plane mirror hung on a wall. The top of the mirror is 6 feet from the floor. Find the least length of mirror required to enable the man to see his whole image in it.

(3) A capital letter A lies on the surface of a table in front of a vertical mirror. Draw a diagram to show the image of the letter in the mirror.

(4) An object placed in front of a plane mirror is moved 10 centimetres farther away from the mirror. What is the change in the distance between the image and the object?

REFLECTION AT SPHERICAL SURFACES

A portion of a spherical surface capable of reflecting light is called a *spherical mirror*. Spherical mirrors are of two kinds: (1) those in which the reflecting surface is concave; (2) those in which the reflecting surface is convex. Let ABC (Fig. 60) be the section of a concave mirror formed as a portion of the inner surface of a sphere whose centre is O. The centre of curvature O of the mirror is the centre of the sphere of whose surface the mirror is a part. The *vertex* C is the middle point of the mirror itself. The *principal axis* OC is a line drawn through the centre of curvature and the vertex. Any other line drawn through the centre of curvature, as HE, is called a *secondary axis*.

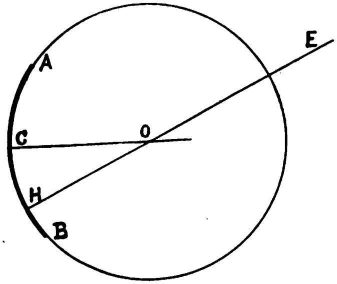

Fig. 60.

A line from any point of a spherical surface to the centre of curvature is a *normal* to the surface at that point. In the case of any curved surface the law of reflection is the same for each point of the surface as it is in the case of a plane surface. A point from which waves of light diverge, or towards which they converge, after reflection is called a *focus*. If the rays parallel to the principal axis fall upon a concave mirror, they will after reflection proceed very nearly to a point F half way between the centre of curvature and the vertex. This point is called the *principal focus*. If the source of light be situated at the centre O, the angles of incidence and reflection are reduced to zero. All rays that pass through the centre of curvature to the mirror will after reflection fall directly back upon themselves, and will be brought to a focus at O.

Let P be a point-source of light waves, and let the spherical surface be concave, with its centre at O (Fig. 61). Draw PC, and at a point near C draw PH.

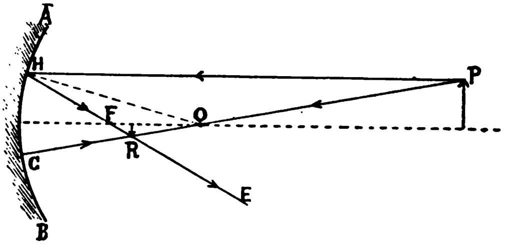

Fig. 61.

Join OH; then, since O is the centre of curvature, OH is a normal to the reflecting surface. Make the angle OHE equal to the angle OHP, letting HE cut the axis CP in R. Similarly, any other ray from P after reflection from the mirror will be found to pass through R, providing the aperture of the mirror be small. If, on the other hand, the luminous point were placed at R, waves diverging from it and reaching the mirror would all be reflected to P. The points P and R are called *con-*

jugate foci. Likewise, if the reflecting surface is convex, waves diverging from P will after reflection appear to diverge from R (Fig. 62). The point R is a virtual focus produced by waves reflected from the surface of a convex mirror. Comparing the two figures, it appears that in the first case R and O are to the right of C, while in the second case they are to the left, the incident rays assumed to travel from right to left. The distinction may be made by representing these opposite directions by opposite signs, calling lines drawn to the right of C positive, those drawn to the left negative. If then the radius of the mirror be r, we have for the concave mirror

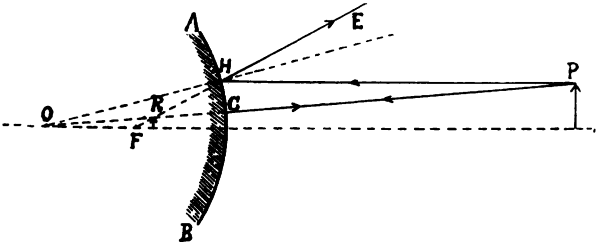

Fig. 62.

$$CO = r,$$

and for the convex mirror

$$CO = -r.$$

In the first figure, since the radius OH bisects the angle PHR,

$$PH : RH = PO : RO.$$

If the aperture of the mirror is small, PC = PH, and RC = RH, very nearly.

Then $$PC : RC = PO : RO.$$

Let $PC = u$, $RC = v$, r being the radius. Also, $PO = u - r$, and $RO = r - v$.

Then $$u : v = u - r : r - v,$$
and $$vr + ur = 2uv.$$
Dividing by uvr,

$$\frac{1}{u} + \frac{1}{v} = \frac{2}{r}.$$

If R is virtual, and v therefore negative, the formula may be written

$$\frac{1}{u} - \frac{1}{v} = \frac{2}{r}.$$

If, as in the case of a convex mirror, both v and r are negative,

$$\frac{1}{u} - \frac{1}{v} = -\frac{2}{r}.$$

To verify the formula connecting the positions of an object and its image in a spherical mirror, and to find the radius of the mirror.

Apparatus: (1) A concave mirror; a convex mirror. (2) An optical bench (Fig. 63). (3) A candle. (4) A small screen made by gumming a square of stiff white paper to the side of a wooden pin about 15 centimetres in height. (5) Two needles.

Errors: (1) The formula holds only in the case of a spherical mirror whose aperture is not large. If a large mirror is used, all but the central portion, about 5 centimetres in diameter, should be covered with a screen. (2) The rays of light proceeding from the object should be near the principal axis of the mirror and as nearly as possible parallel with it. (3) Special care will be required to secure focusing.

Procedure: (I) Support the concave mirror on a holder at the end of the optical bench so that its principal axis shall be parallel to the scale of the bench. Attach the candle to a block sliding on the bench and arrange so that the height of the flame shall be on a level with the centre of the mirror. On another block fix the small screen, and adjust the height of the screen so that it is also on the same level with the centre of the mirror. Place the candle about 100 centimetres from the mirror, and adjust the screen so that the image formed on it may be as distinctly focused as possible. Note whether the image is erect or inverted, also whether it is larger or smaller than the object. Is the image real or virtual? Carefully measure the distance *u* between the object and the mirror and the distance *v* between the image and the mirror. Move the candle nearer to the mirror, again adjust the screen and measure *u* and *v*. Proceed thus to find a series of corresponding values of *u* and *v*, making five trials in all. Note as the candle is moved toward the mirror, the direction in which the image moves. Note also changes as to size, etc., of the image. Find a position in which both object and image are at the same distance from the mirror. In this case $u = v$. Hence

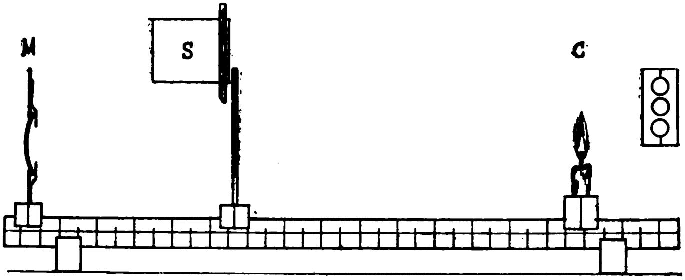

Fig. 63.

$u=v=r$. When the object is at the centre of curvature of the mirror, what is the character of the image formed?

Tabulate as follows:

u	v	$\frac{1}{u}$	$\frac{1}{v}$	$\frac{1}{u}+\frac{1}{v}$	r

Mean value of $r=$ *cm.*

As the candle approaches very close to the mirror, at F and between F and C, can a real image be obtained on the screen? View the image of the candle by looking into the mirror. Is this image real or virtual?

To verify the formula in the case of virtual images the same method will apply, whether the mirror be concave or convex.

(II) Remove a narrow horizontal strip of the silvering from the centre of the convex mirror, thus forming a transparent portion through which a needle can be seen. Support the mirror upon the optical bench. By means of the candle flame determine whether any real image is possible. Place one of the needles not far from the mirror, and place the other behind the mirror so that it can be seen through the glass. Locate the exact position of the image by the method of parallax employed in a preceding experiment. The needle back of the mirror can be made more readily visible by placing a white paper screen behind it. Measure the distance of the object and image from the mirror. Repeat the experiment twice, each time with the object placed in a different position. By the formula calculate the value of r for each separate measurement and find the mean. Tabulate as in the preceding case.

Exercises: (1) Find in a text-book a demonstration proving that the magnifying power of a mirror is the ratio of *u*, the distance of the image from the mirror, to *v*, the distance of the object from the mirror.

(2) How far from a concave mirror of radius 3 feet would you place an object to give an image magnified three times? Would the image be real or virtual?

(3) An object is placed at a distance of 18 inches from a concave mirror 1 foot in radius. Find the position of the image and the magnifying power.

(4) An object is placed half-way between a concave mirror (focal length 30 centimetres) and its principal focus. Find the position of the image. Is the image real or virtual?

(5) Draw a diagram to show the position and size of the image of an arrow placed 10 centimetres in front of a concave mirror whose focal length is 8 centimetres.

REFRACTION OF LIGHT AT A PLANE SURFACE

Whenever light-waves traveling through any medium fall on the polished surface of some transparent medium, a portion of the waves, instead of being reflected back into the first medium, pass on through the second. Let the source of the waves be P (Fig. 64). If the velocity of the waves in the two media is the same, the wave-front will advance in a definite time to the position MON; but, as the second medium is denser than the first, the

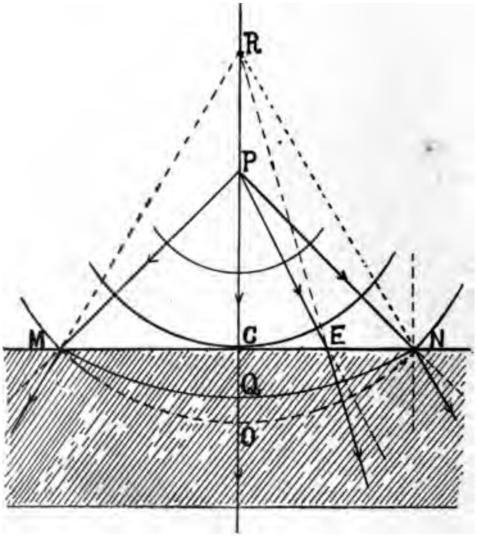

Fig. 64.

wave-front reaches Q instead of O, and the curve of the entering waves is MQN, an arc of a circle whose centre is R, but farther from the surface than P.

Consider also the case where the source P is in the denser medium (Fig. 65). The waves entering the other medium would have the front MON if the velocity were unchanged; but, since the medium is rarer, the actual curve of the entering wave is MQN, an arc of a circle whose centre is R, a point nearer the surface than P. In each of the two cases described R is the virtual image of P.

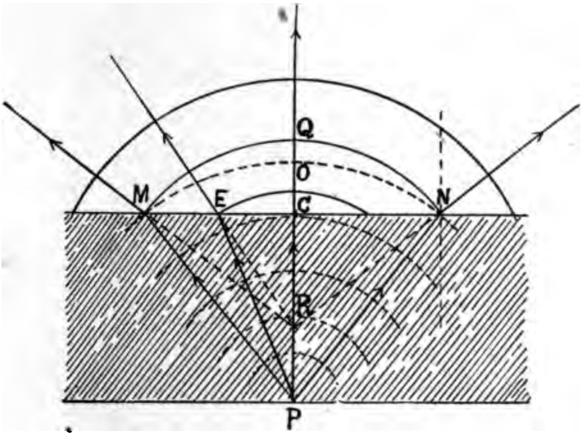

Fig. 65.

Consider, now, the direction of a "ray." If the ray passes through the bounding surface normally, as PC, its direction is not changed. If, however, it falls upon the surface obliquely, as PE, it will be bent. If the second medium is denser than the first, the ray will be bent toward the normal to the surface, while if the second medium is less dense than the first, the ray will be bent away from the normal. A ray whose course is thus changed is said to be *refracted.* Let EC be a ray of light incident at C on a refracting surface, and let MN be the normal at C (Fig. 66). With C as a centre describe a circle of any radius cutting the incident ray at E and the refracted ray at F. Draw ED and FR perpendicular to the normal MN. The ratio of ED to FR is found to be a constant for any two media, whatever be the angle of incidence. This constant is called the *index of refraction* from the first medium into the second. The angle ECD is called the angle of incidence, and the angle FCR the angle of refraction. The ratio ED to CE in the right angled triangle ECD is called the *sine* of the angle ECD. Similarly the ratio FR to CF is the *sine* of the angle FCR.

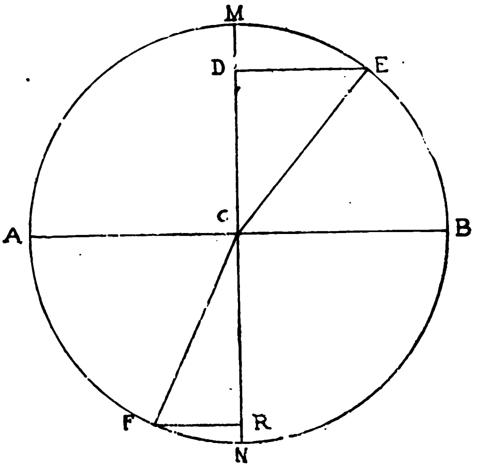

Fig. 66.

$$\text{Sine angle of incidence} = \frac{ED}{CE}.$$

$$\text{Sine angle of refraction} = \frac{FR}{CF}.$$

$$\frac{\text{Sine angle of incidence}}{\text{Sine angle of refraction}} = \frac{ED}{FR}, \text{ since } CF = CE;$$

$$\text{and } \frac{ED}{FR} = \text{the index of refraction.}$$

Hence the index of refraction is defined also as the ratio of the sine of the angle of incidence to the sine of the angle of refraction.

The index of refraction is different for rays of different colors, increasing from the red towards the violet. The refractive indices given in tables are approximately those for yellow light.

To verify the law of refraction at a plane surface, and to determine the index of refraction of glass.

Apparatus: (1) A rectangular block of glass with two opposite sides ground plane and polished sufficiently to allow seeing readily through the whole length of the block. The dimensions of the block may conveniently be 10x7x3 centimetres. (2) A sheet of paper. (3) Some pins. (4) A millimetre scale.

Errors: (1) The refracting surface may not be plane. (2) A somewhat fainter image of the pin due to reflection inside the glass is sometimes confusing. It may be prevented by covering each end of the block with a piece of moist blotting paper.

Procedure: Place the block of glass on the sheet of paper so that its two opposite polished surfaces shall be vertical, and with a pencil mark the outline of the block on the paper (Fig. 67). Set a pin vertically into the paper close to one of the polished surfaces and near one of the vertical edges of the block. With the eye on a level with the table look obliquelv through the glass and notice that the pin seen through the glass is not in the same direction from the eye as that portion of the pin seen over the glass.

Set a pin close to the front surface of the glass so that it exactly covers the image of P. Then place another pin R vertically in the paper so that it appears to be in the same straight line with pin O and the image of pin P. Now remove the glass. Let ABCD represent the outline of the glass on the paper. Then PO shows

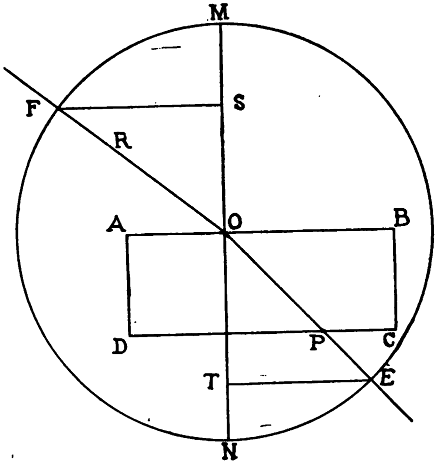

Fig. 67.

the direction of the ray from P through the glass and incident on the surface AB at O. OR shows the course of the ray refracted after it leaves the glass at O. Draw MN normal to the refracting surface. Notice that the refracted ray is bent from the normal in passing from glass into air. Conversely, assuming the ray to be incident in the air along RO, what will be the course of the refracted ray through the glass? Furthermore, what will be the course of the ray after leaving the glass at P and again entering the air? Extend OP and OR. Describe a circle of any radius about O as a centre, meeting OP at E and OR at F. Draw ET and FS perpendicular to the normal MN. Measure ET and FS and calculate their ratio. Since FO = EO,

$$\frac{FS}{ET} = \text{the index of refraction from air to glass.}$$

Change the angle of incidence by placing the pin O at a different point in front of the surface AB, and repeat the measurements. Take measurements for several different incident angles, say five in all. Calculate the index of refraction in each case, and record the results as follows:

FS	ET	$\frac{FS}{ET}$

Mean value of the index,
Value given in table,
Error, Percentage of error,

By extending any two of the adjacent refracted rays backward into the glass, locate approximately the position of the image due to a small pencil of rays proceeding from P and after refraction entering the eye. Is the image real or virtual?

Exercises: (1) Make a diagram showing why in an ordinary plate glass mirror the image is not quite so far behind the reflecting surface as the object is in front. Let P be the object and construct for any two rays, as PA and PB (Fig. 68). Show also where the image would be if produced by the action of the reflecting surface alone.

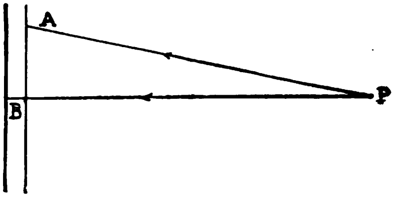

Fig. 68.

(2) Draw a diagram to show the real and apparent positions of an object viewed obliquely at the bottom of a pond.

(3) Prove by geometry that the emergent ray is parallel to the incident ray after passing through a glass plate with parallel sides.

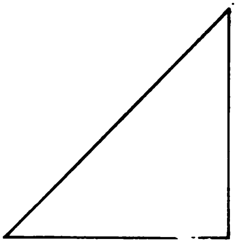

Fig. 69.

(4) Show how light could traverse without refraction a prism with the form of cross-section shown in Fig. 69. State what is meant by *total reflection* and *critical angle*. If the material composing the prism had a critical angle greater than 45°, would the light then traverse the prism without refraction?

REFRACTION THROUGH A LENS

A lens is a portion of a transparent medium bounded by two curved surfaces, usually spherical. A straight line drawn through the centres of the two spheres whose surfaces bound the lens is called the *principal axis* of the lens. Lenses are divided into two classes, according to their action on a parallel beam of light: (1) *converging lenses,* which are thickest at the axis, as in the type called "double convex"; (2) *diverging lenses,* which are thinnest at the axis, as in the type called "double concave." Rays of light falling on a convex lens in a direction parallel to the principal axis will after refraction converge toward a point on the axis. This point is called the *principal focus* of the lens Rays falling on a concave lens in a direction parallel to the principal axis will be made to diverge, and will appear to come from a point on the axis on that side of the lens toward which the incident rays proceed. This point is the principal focus of the concave lens. The distance between the lens and its principal focus is called the *focal length* of the lens. The *optical centre* of a lens is a point on its principal axis, either at or near its centre of volume, so situated that a ray passing through this point and the lens suffers no change of direction. That is, for any ray passing through the optical centre, the emergent ray is parallel to the incident ray, but suffers a lateral displacement depending on the angle of incidence and the thickness of the lens. The displacement of a ray passing through the optical centre of a thin lens is so slight that it may be neglected in constructions. Any straight line drawn through the optical centre of a lens is called an *axis*. Lines drawn from a lens in a direction opposite to that in which the incident rays travel are called positive; lines drawn from a lens in the same direction as that in which the incident rays travel are negative. Thus (Figs. 70 and 71) the focal length OF is negative in the case of the convex lens, positive in the case of the concave lens. In all cases in which the principal focus is real the focal length is negative.

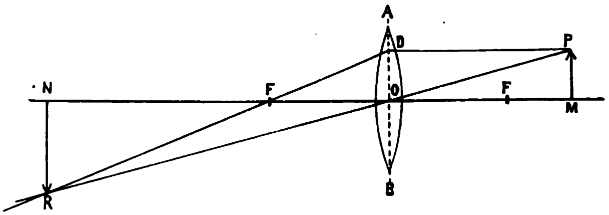

Fig. 70.

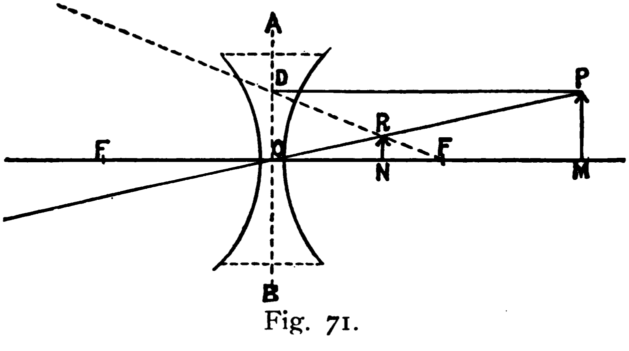

Fig. 71.

Given a source of light at a point P on one side of a converging lens, waves falling upon the lens from this point will be refracted, and will converge to a point R on the other side. Conversely, if R be the source of light, the waves will be brought to a focus at P. The points P and R are called *conjugate foci.*

In determining by construction the position of the image formed by a lens, three principles are considered:

(1) Two rays diverging from a point toward a lens will represent all rays from that point in the formation of the image.

(2) A ray falling on a lens in a direction parallel to the principal axis passes through the principal focus after refraction.

(3) A ray incident at the optical centre passes through the lens with its direction unchanged.

Consider the case of the double concave lens:

Let $OM = u$, $ON = v$, $OF = f$. By construction, $DO = PM$.

Then $\frac{u}{v} = \frac{OM}{ON} = \frac{PM}{RN} = \frac{DO}{RN} = \frac{OF}{FN} = \frac{f}{f-v}$,

and $$uf - vf = uv.$$

Dividing by ufv, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$.

For a convex lens the focal length is negative. There are two cases:

(1) When the object is inside of F and the image virtual, the formula can readily be obtained by a method of proof similar to that just given. Or, since f is negative, the above formula becomes

$$\frac{1}{v} - \frac{1}{u} = -\frac{1}{f}.$$

(2) When the object is beyond F and the image real, v and f are both negative, and the formula becomes

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}.$$

Again, as in the case of the spherical mirror, it is evident by geometry that the magnifying power for the lens is the ratio of v to u.

To verify the relation between the positions of an object and its image formed by a lens, and to determine the focal length of the lens.

Apparatus: (1) An optical bench. (2) A convex lens. (3) A half-convex lens, or a half concave lens. (4) A gas-burner. A small kerosene lamp may be used. (5) A metal screen with a narrow slit opening about 5×20 millimetres cut through its middle. (6) A small screen of white card-board. (7) Some stout pins. The lens and screens should be supported on blocks made to slide along the optical bench.

Errors: (1) It is important that the principal axis of the lens should lie parallel with the line joining the centres of object and image and be as near to that line as possible. (2) The outline of the image may be indistinct, due principally to aberration.

Procedure: (I) At one end of the optical bench place the gas-flame. Move the convex lens to such a position on the bench that a real image of the flame is seen upon the screen when suitably placed. Arrange the metal screen in front of the flame so that the centre of the illuminated slit and the centre of the screen shall be near the axis of the lens. Adjust the screen so that the image of the slit is as sharp as possible. Measure the distance u of the slit from the lens, also the distance v of the screen from the lens. Repeat the measurements several times, each time diminishing the distance between the flame and the screen. At each change in the relative distances, note the size of the image as compared with the object. Is the image erect or inverted? Calculate the value of f from the three measurements, using the formula in the form

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}.$$

Record the results as follows:

u	v	$\frac{1}{u}$	$\frac{1}{v}$	$\frac{1}{u}+\frac{1}{v}$	f

Do the results of the measurements, when taken together, verify the formula for the lens? If so, in what manner?

State the law of the convex lens in words.

Move the lens close to the illuminated slit. If no real image can be formed upon the screen, where must the eye be placed in order that an image can be seen? Again note whether the image is real or virtual, magnified or diminished, erect or inverted.

(II) Replace the cardboard screen by a pin set upright in the block. Move the pin to a position about 30 centimetres from the end of the rod of the optical bench. Place the lens farther away from the same end of the rod. Support the apparatus so that, on looking through the lens, an image of some distant object can be seen. Looking sharply at the pin, adjust the lens until the image, formed in air, coincides with the pin. Test the adjustment by the method of parallax. Move the head about in different directions. If the pin is more distant from the eye than the image, it will appear to move past the image in the same direction as the eye moves. If the pin is nearer than the image, it will appear to move past the image in the direction opposite to that in which the eye moves. When the pin and image coincide, there will be no apparent motion of either one with respect to the other when the eye is moved in any direction. Measure the distance from the lens to the pin. Repeat the trial several times and compare the mean of the results with the mean value of f found in (I).

(III) Take a half-lens, either concave or convex, and mount it so that its edge is vertical. Hold a pin behind the lens in such a position that its virtual image can be seen. Place the pin so that its point is opposite the middle of the lens at the edge and the pin itself horizontal and perpendicular to the axis. It is now possible, while looking along the axis of the lens, to hold a second pin near the edge of the lens so that its point seems to coincide with the point of the image. By measuring the distances from the lens to the pins, values of u and v are obtained, which, by using the appropriate formula, give the value of f. Repeat the trial twice and find the mean of the results. State in what manner the results obtained go to verify the formula for the lens.

Exercises: (1) The focal length of a convex lens is 20 centimetres. Find the conjugate focus for a point 30 centimetres from the lens.

(2) If the distance of an object from a convex lens is twice the focal length of the lens, what will be the position of the image? What will be the size of the image compared with the object?

(3) If an observer's eye be held close to a convex lens of 3 centimetres focal length to view an object at a distance of 2.5 centimetres from the lens, show that the magnifying power is 6.

(4) Show the advantage of a long-focus lens over one of short focus in the magnification of a very distant object.

ELECTROSTATIC INDUCTION

When any charged body, however electrified, is brought near an insulated unelectrified conductor, this conductor becomes electrified, a dissimilar charge appearing on the side nearer the electrified conductor and a similar charge upon the farther side. A conductor charged in this way, by the presence of an electrified body and without contact, is said to be electrified by *induction*. On removing the inducing charge, the two unlike charges induced in the insulated conductor mutually destroy one another, hence all signs of induced electrification disappear.

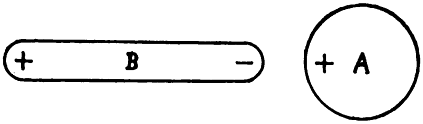

Fig. 72.

Let a body A charged positively be placed near an insulated conductor B (Fig. 72). If B be connected to earth while under the influence of A, the repelled positive charge in B will pass off, leaving only the attracted negative charge. This latter charge is said to be "bound," while the charge which goes to the earth is "free." If now A be removed while B remains insulated, the charge on the latter is uniformly distributed. B is said to have been charged by induction.

The study of the phenomena of electrostatic induction is aided by the use of the *electroscope*. This is an instrument used to detect the presence of an electric charge, and to determine whether the charge is positive or negative. A simple form of electroscope, called the gold-leaf electroscope, is shown in Fig. 73. It consists of two strips of gold leaf enclosed in a glass vessel and attached to the lower end of a metal rod. The outer end of the rod, which projects into the open air, terminates in a metal plate. The rod is insulated where it passes through the neck of the vessel.

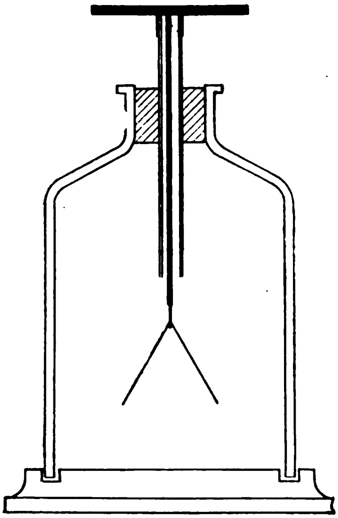

Fig. 73.

If the plate of the electroscope is touched by an electrified body, part of the electrification will go to the gold leaves, and as these are charged with the same kind of electrification, they will be repelled from each other. In this case the electrification on the gold leaves is of the same sign as that on the electrified body. When the charged body does not touch the plate but is held near to it, the conditions become essentially the same as those described in the preceding paragraph. The metal parts of the electroscope constitute the insulated conductor; the plate, being the part nearest the charged body, will have the opposite electrification to that body; while the gold leaves, being the parts farthest from the charged body, will have the same kind of electrification as that body.

To study electrostatic induction by the aid of the gold-leaf electroscope.

Apparatus: (1) A gold-leaf electroscope. (2) Rods of glass and of ebonite. (3) Pieces of fur and silk.

Errors: It is essential that all the apparatus should be dry. A moist atmosphere will interfere with results. It may be necessary to warm the rods near a fire or in an air bath in order to obtain satisfactory electrification.

Procedure: (*a*) Electrify a glass rod by rubbing it with silk. Bring it into contact with the plate of the electroscope for an instant, then remove it. Bring the electrified glass rod near the plate, then remove it again. Note what takes place. Discharge the electroscope by touching the plate with the finger.

(*b*) Charge the electroscope again by bringing an electrified glass rod into contact with the plate. Electrify an ebonite rod by rubbing it with fur. Bring it gradually toward but not touching the plate, then withdraw it. Repeat the trial several times, noting carefully the action of the leaves. Discharge the electroscope.

(*c*) Bring an ebonite rod, or a glass rod, more strongly electrified than before, gradually towards but not touching the plate of the electroscope, then withdraw it. Repeat and watch the leaves. At each trial does the charge in the electroscope appear to be permanent or temporary? Compare the observations with those of (*a*) and (*b*).

(*d*) Bring a charged ebonite rod near enough to the plate to produce a marked divergence of the leaves.

Touch the plate with the finger while holding the rod still. Remove, first the rod, then the finger. Explain the result.

(*e*) Again bring the charged ebonite rod near the plate of the electroscope. Touch the plate with the finger; but in this case remove first the finger, then the rod. Is the effect upon the leaves now temporary or permanent? Explain and compare with the result obtained in (*d*). Make sure that the electroscope is discharged before proceeding with the next.

(*f*) Charge the electroscope permanently by induction, using an ebonite rod rubbed with fur. Bring the palm of the hand down toward the plate. Explain the action of the leaves. Discharge and charge again, using the ebonite rod as before. Bring the charged ebonite rod slowly into the vicinity of the plate and watch the movement of the leaves during the slow approach of the rod, also during the slow withdrawal of the rod. The electroscope being still charged repeat the operation, using a glass rod electrified by rubbing with silk. Observe the action of the leaves, and compare with the results obtained when the charged ebonite rod was brought into the neighborhood of the charged electroscope.

Exercises: (1) Three insulated metal balls, A, B, and C, are placed in a line, A and B in contact, C a little way off. C is positively electrified, and then A and B are separated. What are now the electrical states of A and B? Represent fully by means of drawings.

(2) An insulated conductor A is brought near to the plate of a gold-leaf electroscope charged positively. Explain what will happen (*a*) if A is unelectrified, (*b*) if A is charged positively, (*c*) if A is charged negatively.

(3) Show how one may determine the kind of charge existing in an electrified body.

CAPACITY OF A CONDUCTOR

When two insulated conductors are brought near together and one is charged, the other becomes charged by induction. The medium through which the induction takes place is called the *dielectric*. Let A be a metal plate charged positively (Fig. 74). Bring near A an uncharged insulated conductor B, such as another metal plate; charges are induced on B, negative on the nearer side, positive on the farther side. These opposite charges come as close together as possible by collecting on the faces nearest each other; nearly all the *strain* is in the medium between the two conductors. Let B be connected to earth. The negative charge on B attracts a large part of the positive charge on A and holds it "bound," so that there is a smaller amount of free charge on A. While the total amount of charge has not been diminished, the strain at A has been so lowered by B that an additional positive charge can be given to A. This new charge will induce more negative charge on B, which in turn will react on A, binding more positive charge on the side opposite B. By this mutual action between the two conductors, A is rendered capable of holding a greater amount of positive charge than if it stood by itself. In other words, the *capacity* of A has been increased by the presence of B.

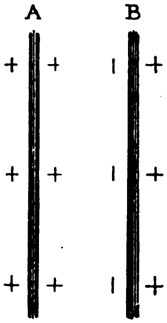

Fig. 74.

In a similar manner, if A is charged negatively, its capacity may be increased by bringing it near B, joined to the earth and separated from A by a dielectric.

Different dielectrics possess different powers of transmitting induction. That property of the dielectric with reference to the extent to which induction takes place through it is termed its *specific inductive capacity.* The inductive capacity of air is taken as the standard, and the specific inductive capacity of any given substance is expressed in terms of this. Thus, solid paraffin has a specific inductive capacity of 2.29, ebonite of 3.15, sulphur of 3.97, and dense flint glass of 7.37.

To study some conditions affecting the capacity of a conductor.

Apparatus: (1) A pair of insulated metal plates. Each plate may be of sheet zinc cut about 15×15 centimetres square and attached to wooden blocks in the manner shown in the figure below. (2) A gold leaf electroscope. (3) A glass plate about twenty centimetres square. This can be supported vertically by inserting one corner at its lower edge in a groove cut in the side of a block. (4) Some fine wire. (5) Ebonite rod and fur, or glass rod and silk.

Errors: Same as in the preceding experiment. Guard against leakages as much as possible.

Procedure: (*a*) Attach one of the insulated zinc plates A to the plate of the electroscope by means of a fine wire (Fig. 75). Keep the free plate B connected to the earth by means of the hand. Remove the plate B to some distance. Apply a charged rod to the surface of A until the leaves of the electroscope show a strong divergence.

The degree of divergence of the leaves is an indication of the extent to which the metal parts of the electroscope and the zinc plate, together forming one conductor, have been charged. Now bring the free plate B towards A and note the effect upon the leaves. Consider fully all the effects that were produced by bringing the plates near together. Has a discharge taken place? Has the charge on A been changed? Apply more charge to A. Continue applying charges to A, and note whether the leaves can be made to diverge as much as at first. What has been the effect of B upon the capacity of A? What has been necessary in order to maintain the strain at A near a constant value?

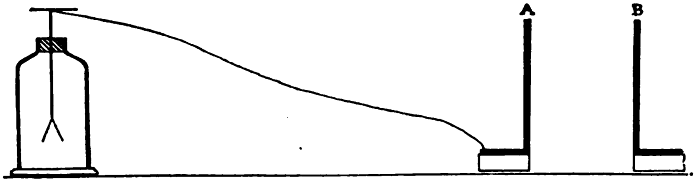

Fig. 75.

(*b*) Remove B to some distance from A. Mount the glass plate vertically in front of, and in contact with, A. Charge A again, and touch B so as to bring its charge to zero. Move up B until it touches the glass, and note the effect upon the leaves of the electroscope. Withdraw B, keeping its surface opposite the surface of the glass plate. Explain the effects, and compare with those noted in (*a*).

(*c*) Arrange the two zinc plates opposite each other with the glass plate between them. Attach A by means of a wire to the electroscope. Charge A, and touch B to bring its charge to zero. Charge again, and note the divergence of the leaves. Withdraw the glass plate suddenly. Fxplain the significance of the effect noted. Repeat the experiment several times, if necessary, until the results are satisfactory and are clearly understood.

Exercises: (1) Two conductors, A and B, are insulated and are placed near to one another. (*a*) A has a charge + V. What will be the condition of B? (*b*) If the charge on A be increased to a sufficient amount, it is observed that at last a spark will pass between B and A. What does this prove as to the change in the condition of B during the charging of A?

(2) What is a condenser? Give a simple illustration.

(3) Can a Leyden jar be charged heavily when it stands on a glass plate? Explain fully.

THE MAGNETIC FIELD OF FORCE

The region surrounding a magnet, in which it produces a stress, is called a *magnetic field.* The ether stress about the magnet is of such a nature that a single magnetic pole tends to move in a fixed direction and a freely suspended magnet takes up a definite position. We may imagine lines drawn at various parts of a magnetic field, each indicating the direction in which a single magnetic pole would move. Such lines are called *magnetic lines of force.* If a very small magnet is placed at any point in a magnetic field, it will place itself tangent to the line of force at that point. The positive direction of a line of force is assumed to be that in which a north pole would be urged if placed on the line. The lines of force may, therefore, be regarded as passing from the north end of a magnet to the south end, continuing through the magnet, forming complete closed circuits.

To plot the field of a bar magnet.

Apparatus: (1) A bar magnet. (2) A large sheet of paper. (3) A small pocket compass. (4) A thread about one metre long. (5) A pair of wooden blocks. (6) Some sheets of good quality blue-print paper cut the size of the note-book. (7) Iron filings. Filings made with a clean file from soft Norway iron are the best. (8) A small sieve or pepper box.

Errors: (1) The space around the magnet may not be free from the presence of masses of iron. (2) Owing to the size of the compass case, it is difficult to mark accurately the position of the end of the needle. (3) The blue-print paper used in (II) should be kept in a dark place until the rest of the apparatus is arranged.

Procedure: (I) Place the magnet in the middle of the sheet of paper on a table (Fig. 76). Draw a pencil line around the magnet to mark its exact position. Inside of this outline mark near one end the letter N, corresponding with the north pole of the magnet, and near the other end the letter S, corresponding with the south pole. Remove the magnet to a distance of several feet from the paper. Place the compass on the paper and notice the direction in which the needle points when no magnets are near it. The vertical plane in which the axis of the needle lies is called the "magnetic meridian," and the directions in which the ends of the needle point are respectively, "magnetic north" and "magnetic south,"

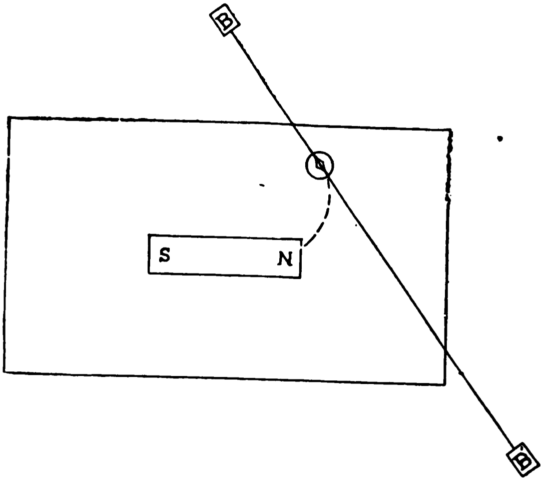

Fig. 76.

By means of thumb-tacks fasten the ends of the thread to the two blocks placed one on each side of the paper. Move the blocks until the thread is stretched directly over the ends of the needle and so lies in the meridian. Mark off twenty points on the outline of the magnet as starting-points in plotting the lines in the field. Six points should be placed much closer together around each end of the magnet, and the remaining eight distributed equally along the sides. Return the magnet to its place on the paper. Place the compass near one of the starting points. Then if there were no magnetic forces acting, save those due to the poles of the magnet, the needle would take up a position which is the resultant of the forces due to the two poles. As the influence of the earth on the needle is not small compared to that of the magnet, the sheet of paper should be turned so that the needle is in the meridian marked by the thread. Care should be taken that the needle does not turn away from the point selected as the beginning of the line. Repeat the adjustment, if necessary, until the desired starting-point and the needle both lie in the same meridian. Mark close to the compass case the position of the end of the needle away from the magnet. Move the compass so that the end of the needle nearest the magnet is over the point just marked. Turn the paper so that the point last marked and the direction of the needle are both brought under the thread. Mark the position of the other end of the needle as before. Continue moving the compass in this way until the edge of the paper is reached or until the compass returns to the magnet. A smooth curve drawn through the points marked will represent a line of force. Indicate by an arrow the direction in which a north pole would move along the line. Proceed in the same manner to plot the lines starting at the other points marked around the magnet. Make a reduced copy of the result in the note-book, showing the lines of force in the magnet's field.

(II) Iron filings make admirable exploring needles, because they are so short. By placing a magnet under a sheet of paper or glass and sprinkling iron filings loosely over the surface, each minute bit of iron becomes a magnet by induction, and so will take the direction of the line of force. The magnet must be held or placed firmly so it will not move during the manipulation. The filings should be scattered *evenly* and *thinly* over the sheet and made to arrange themselves by tapping the sheet lightly with a pencil. If the sheet is then carefully lifted vertically out of the field, the filings will not be disturbed.

A permanent record of the lines of force may be obtained by allowing the filings to arrange themselves upon a sheet of blue-print paper. Lay the magnet on the table, and over it place a sheet of blue-print paper, colored side up, using two blocks of wood of the same thickness as the magnet to support the paper. Secure the paper, if necessary, by means of non-magnetic weights. With the sieve scatter the iron filings evenly in a thin layer on the paper, allowing them to fall from a height of 20 to 30 centimetres. Tap the paper lightly with a pencil until the outline of the magnet and the lines of iron filings are distinct. Allow the paper to remain in position from 5 to 15 minutes, according to the light, or until the uncovered portions of the paper have changed color considerably. Then run off the iron filings and thoroughly wash the paper in a bath of cold water. Hang the paper up to dry. When dry a permanent print remains and the sheet may be inserted in the note-book.

In this manner obtain a copy of the lines of force in the following:

(1) A complete bar magnet.
(2) Two magnets with like poles near together.
(3) Two magnets with unlike poles near together.
(4) Four poles arranged as shown by the letters.

N

S S

N

(5) Four poles, as in (4), with an iron ring in the centre.

Exercises: (1) Draw a diagram showing lines of force from one pole of a bar magnet to the other pole.

(2) Two bar magnets are laid in the same straight line with their N poles adjoining, but separated by a short distance. Draw a diagram to show the lines of force between these poles.

(3) Also draw a diagram to show the lines of force between a N pole of one magnet and an adjacent S pole of another in the same line.

(4) Draw the lines in a field about a N pole in a plane *perpendicular* to the axis of the magnet.

(5) Show by a sketch the lines of force in a field due to a single isolated magnetic pole.

THE MAGNETIC FIELD ABOUT AN ELECTRIC CURRENT

Assume two bodies A and B oppositely electrified, A having a positive charge and B, a negative charge. When the two bodies are joined by a conductor, the strain in the space between the bodies disappears, and the conductor is said to be conveying an "electric current." The current will be temporary, if, as in the case of the Leyden jar, the quantity of electrification is limited. If, however, the charges are replaced as fast as they disappear, by connecting A and B with the terminals of a Voltaic battery, the flow of current will be continuous. It is convenient to think of the current as being a flow from the body positively charged to the other. In all ordinary cells, in which zinc is one of the elements, the zinc terminal is found to be negatively electrified and the other terminal positively electrified. The direction of the current is, therefore, considered as being along the wire from copper to zinc, and in the liquid, from zinc to copper. As the process continues, the following phenomena will occur: (*a*) the zinc dissolves away, while bubbles of hydrogen gas collect on the other plate and rise to the surface; (*b*) the temperature of the wire is raised, showing that the process of solution of zinc in acid furnishes energy; (*c*) a magnetic field will be produced about the wire. The lines of force in the magnetic field have a definite relation; first, with respect to the direction of the current, and, second, with respect to the wire.

To study the lines of force in the magnetic field associated with a current.

Apparatus: (1) A single battery-cell. (2) A magnetic compass. (3) About one metre of insulated fine copper wire. (4) Connectors. (5) A wooden block on which to support the compass.

Errors: Care must be taken during the observations that good electrical connections are maintained, and that the cell is in proper condition.

Procedure: (*a*) *Horizontal Current.* Connect the ends of the long wire to the terminals of the cell. Place the compass on a block and at some distance from the cell, then turn the compass box till the north-seeking pole of the needle is over the letter N of the dial. Straighten out about one-third of the wire and hold it in the magnetic meridian at some distance above the compass and so that the current which flows toward the zinc terminal shall

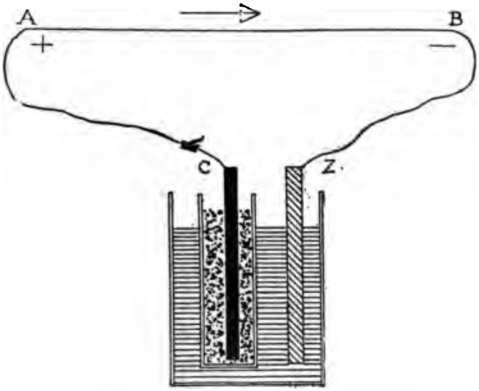

Fig. 77.

flow from south to north. Now slowly bring this portion of the wire down toward the needle and notice the relation between the deflection produced and the distance of the wire from the needle. Also notice the direction in which the north pole of the needle turns. Rotate the wire slowly in the horizontal plane and observe the position taken by the needle when the wire lies in an east and west direction. When the wire is brought around again into the meridian, and the direction of the current reversed, note the direction in which the north pole of the needle swings. Show these relations between wire, direction of current, and needle by means of a drawing after the manner of the one here given (Fig. 78). Mark the north pole of the needle N. Hold the wire near either side of the compass box, in the horizontal plane of the needle and parallel to the magnetic meridian. Note whether the current now appears to have any effect on the needle. But for the force due to the earth's field, the needle would always point directly at right angles to the wire which carries the current. On account of the earth's magnetism, however, it comes to rest in the position where the forces of the earth's field are balanced by those due to the current. Place the wire in a groove cut across a block of wood, and arrange so that the current flows from south to north. Hold the compass in the hand so that the north pole of the needle is directly over the mark N of the dial. Bring the compass slowly down toward the wire and notice the direction of deflection of the needle. Rotate the

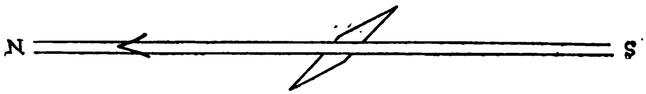

Fig. 78.

wire, keeping it in the groove in the block, without changing the position of the compass. Observe the effects produced when the conductor extends east and west, and also when the current is reversed. Show the results of the observation by means of drawings.

(*b*) *Vertical Current.* Support the compass on a block at some distance from the cell. Hold a straight portion of the wire in a vertical position at some distance in front of the north pole of the needle and so that the current flows up. Reverse the current and note the change in the deflection. Repeat the observations at the south pole of the needle. Represent the results by means of drawings similar to that in Fig. 79. Show the direction of the current along the wire. Repeat all observations in the following manner:

At each trial let one pupil raise the zinc element from the battery, thus breaking the circuit, while the other pupil adjusts the wire near the compass in the position desired. Then quickly lower the zinc into the cell and note the effect on the needle.

Combine the results of all observations to form a general conclusion with respect to the direction of the line of force associated with a current. Suppose the direction of the current to be that of the forward motion of a right-handed screw as it enters the nut, how would the direction of the magnetic force on the positive pole compare with the direction in which the screw turns? State fully and show by a plain drawing like the one given in Fig. 80. Show the direction

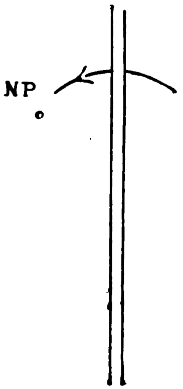

Fig. 79.

of the current along the wire.

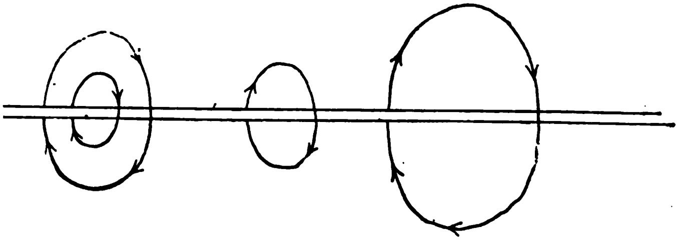

Fig. 80.

(*c*) *Multiple Currents through Coils.* Make a rectangular loop by winding the wire once around the block. Hold this loop in a vertical plane in the meridian. Move the compass from either side toward the loop and notice the deflection produced as it approaches the centre of the loop. Increase the number of windings in the coil and repeat the observations. Rotate the loop in a vertical plane and observe the effects when the plane of the loop is east and west and when it is again in the meridian but the direction of the current reversed. Make a drawing (Fig. 81) showing a single wire loop carrying a current, and

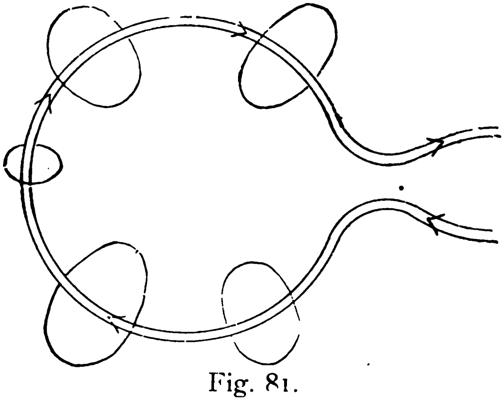

Fig. 81.

indicate the direction of the lines of force at different parts of the loop.

(*d*) *Polarity of a Coil.* Make a coil with several windings, and trace the direction of the current through it. Holding the plane of the coil vertical, bring one face toward the N-pole of the needle, then toward the S-pole. Note the effect in each case, and explain. Repeat the observations, using the other face of the coil. From which face of the coil do the lines of force come out? At which face do they enter the coil? Which face of the coil is N-pole and which is S-pole? Show by means of a drawing, indicating in the usual manner the directions of the lines of force. Show the poles by letters N and S.

Exercises: (1) A wire is stretched north and south, and a current flows from south to north in the wire; describe the position taken by a compass needle when placed (*a*) above the wire, (*b*) under the wire.

(2) A current is flowing along a vertical rod which passes through a hole in a horizontal card. Iron filings are sprinkled on the card in the neighborhood of the rod. How will the filings arrange themselves?

(3) A coil of wire is suspended in front of the N pole of a bar magnet. A current is made to flow in the coil. State precisely the position that the coil will take with reference to the magnet.

MEASUREMENT OF CURRENT

THE TANGENT GALVANOMETER

It has been seen that if a conductor through which a current is passing forms a loop, the space within it possesses magnetic properties. All the lines of force passing through the loop urge the N-seeking pole of a needle in the same direction. Consider the strength of the current measured in absolute electromagnetic units.

If a conductor one centimetre long be bent into an arc of one centimetre radius, the current through it will have unit strength when it exerts a force of one dyne on a magnetic pole of unit strength placed at the centre of the arc. If the length of the arc be l, the force will be l times as great; if the radius be r, the force will be $\frac{l}{r^2}$, since the force varies inversely as the square of the distance. If the conductor make a complete circle, the force will be $\frac{2\pi r}{r^2}$, and if there be n circular coils of the same radius r, the force will be $\frac{2\pi rn}{r^2} = \frac{2\pi n}{r}$. This force exerted by unit current upon unit pole is denoted by G, and is called the *constant of the coil.* If a current of strength C flows through the coil, the force is C times as great. Then if the magnetic pole is of strength m, the current C will exert a force

$$Cm\frac{2\pi n}{r}, \text{ or } GCm.$$

The direction of this magnetic force at the centre is, of course, perpendicular to the plane of the coil.

If a coil of large radius is placed with its plane in the magnetic meridian, and a small magnetic needle hung at its centre, the arrangement constitutes a *tangent galvanometer* (Fig. 82). The magnetic field produced by the current through the large coil is nearly uniform near its centre, and is perpendicular to the plane of the coil. The needle will be deflected, and will come to rest at an angle such that the moments of the forces due to the fields of the earth and the current balance each other.

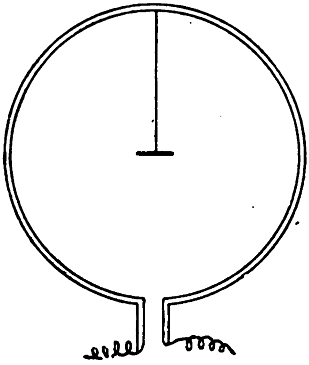

Fig. 82.

Let NS be the magnetic meridian in which lies the plane of the coil with its centre at O (Fig. 83). Let H be the force of the earth's field acting on a unit pole, then the force acting on a pole of strength m is Hm; the force due to the current C flowing through the coil is GCm. These forces act at right angles to each other, and since the needle is free to rotate, it will turn into the direction of the resultant force FD, making an angle of deflection a. The ratio ED to EF in the right triangle EFD is called the *tangent* of the angle EFD, and is written

$$\tan \mathrm{EFD} = \frac{\mathrm{ED}}{\mathrm{EF}}.$$

But, since in the figure the angle EFD measures the deflection a, and since ED is the measure of the component GCm and FE the measure of the component Hm,

$$\tan a = \frac{GCm}{Hm}.$$

Whence

$$C = \frac{H}{G} \tan a.$$

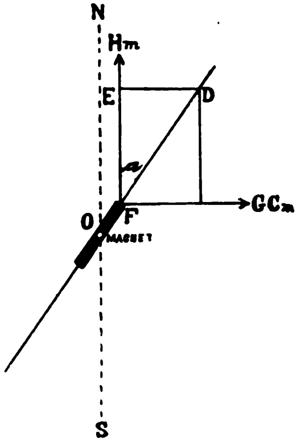

Fig. 83.

The formula gives the value of the current C in absolute units The practical unit of current, the *ampere*, is equal to one-tenth of an absolute unit. Hence in amperes,

$$C = 10 \frac{H}{G} \tan a.$$

The expression $10\frac{H}{G}$ is called the *reduction factor* of the galvanometer and is denoted by letter K. The value of the current in amperes then may be written

$$C = K \tan a.$$

Whence

$$K = \frac{C}{\tan a}.$$

In conclusion, the above discussion may be summed up as follows:

(1) The strength of a current is proportional to the tangent of the angle of deflection.

(2) The strength of a current is found by multiplying the tangent of the angle of deflection by a constant whose value depends on the earth's magnetic force, the dimensions of the coil, and the number of turns in the coil.

In using a tangent galvanometer:

(1) Place the instrument where it will be as free as

possible from the influence of magnets or magnetic substances.

(2) In placing the instrument, see that it is level, and that the plane of the coil is in the magnetic meridian. The ends of the pointer attached to the needle should be at zero.

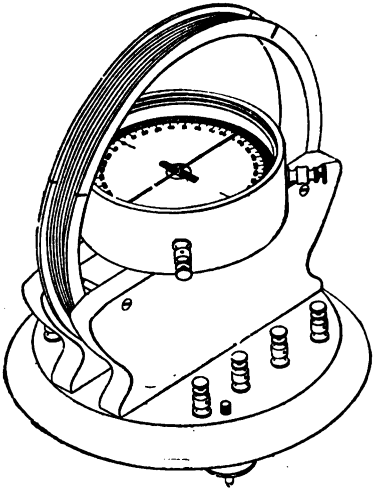

Fig. 84.

(3) Both ends of the pointer should always be read to correct for possble errors due to an imperfect mounting of the needle.

(4) Guard against the zero error by noting carefully before each deflection of the needle the exact positions of the ends of the pointer. Make the necessary correction to obtain the true value of the deflection.

(5) Take double readings; that is, read both ends of the pointer, then reverse the current, thus reversing the deflection, and read again. The mean of the four readings should be taken as the true deflection.

(6) Avoid parallax. Place the eye above the needle, so that the pointer hides its own image in a mirror placed beneath.

(7) If the needle is poised on a pivot point, tap the galvanometer gently before reading. This will assist the needle in settling to its correct position.

(8) The wires leading to and from the galvanometer should be twisted together, so that the current flowing through them may not exert a direct influence upon the needle.

(9) It is important in using a tangent galvanometer to consider the best values of deflections to use. The following data, taken from Stewart and Gee, Vol. II, p. 230, will explain: "On consulting a table of tangents it will be found that the effect of making an error in the reading will be greater at the extremities of the scale than at the middle, thus,

tan 10° = .1763	tan 45° = 1.000	tan 80° = 5.671
11° = .1944	46° = 1.0355	81° = 6.314
Difference .0181	Difference .0355	Difference 0.643
= 9.8 per cent.	= 3.5 per cent.	= 10.7 per cent.

of the whole mean effect."

Arrange the conditions of the experiment, by varying the resistance in the circuit, so that the deflections shall be in the neighborhood of 45°. This can generally be accomplished by varying the number of turns in the galvanometer coil.

THE HYDROGEN VOLTAMETER

When an electric current is made to decompose an electrolyte it is found that the mass of gas liberated, or the mass of metal deposited, is proportional to the strength of the current. The mass of a substance liberated in one second by a current of one ampere is termed its *electro-chemical equivalent.* Thus in one second an ampere of current will deposit from silver nitrate .001118 grammes of silver and will liberate from water .00001038 grammes of hydrogen. Let W denote the mass in grammes of a substance liberated at one electrode by a current of strength C in t seconds, and let e be the electro-chemical equivalent. Then

$$W = Cet\text{, or } C = \frac{W}{et}.$$

A simple form of hydrogen voltameter is shown in Fig. 85. The acidulated water is placed in the vessel A two platinum electrodes are cemented into the base of the

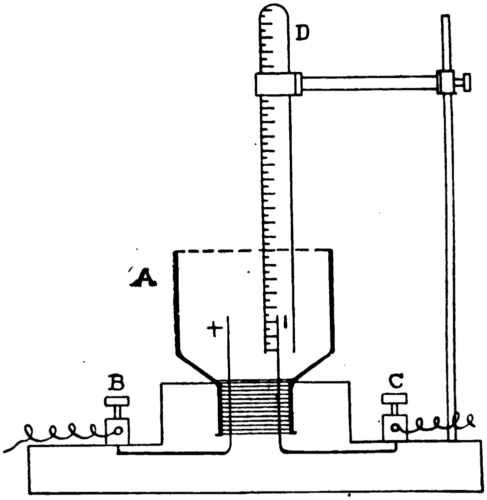

Fig. 85.

vessel and are connected to the binding posts B and C, at which the current is made to enter and leave the voltameter. The graduated tube D is filled with the acidulated water and inverted over the negative electrode. The hydrogen is collected in D, while the oxygen is liberated into the air at the other electrode.

Suppose the gas liberated in a certain time has the apparent volume V. To obtain the true volume several corrections must be made, chief of which are those for temperature and pressure. (See Fig. 86.)

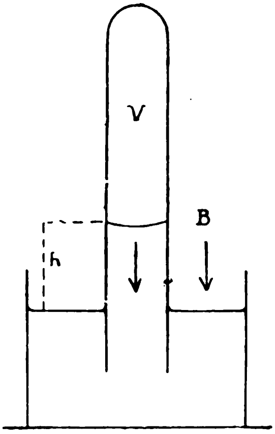

Fig. 86.

The observed volume V exists at the pressure B, as given by the barometer, less the pressure due to the height h of liquid that still remains above the level of the liquid in the vessel. The pressure due to h must be reduced to centimetres of mercury in order that it may be expressed in the same units as the barometric pressure. Calling d the relative density of the liquid, then $\frac{dh}{13.6}$ will be the equivalent pressure in centimetres of mercury. The corrected pressure P then becomes

$$P = B - \frac{dh}{13.6}.$$

Let T be the observed temperature of the gas expressed in the absolute scale. Then if the standard temperature (273°C., absolute) be T′ and the standard pressure (76 cm.) be P′, the volume of the gas V′, reduced to N. P. T., can be found from the relations given in the following:

$$\frac{PV}{T} = \frac{P'V'}{T'}.$$

The true volume of the gas being thus found in cubic centimetres, the current in amperes will be, when the gas is hydrogen,

$$C = \frac{V'}{t \times .1156}.$$

where *t* denotes the time in seconds during which the gas was liberated. A current of one ampere flowing for one second liberates 0.1156 cubic centimetres of hydrogen at 0°C. and 760 millimetres pressure.

THE DANIELL CELL

A form of standard cell which is commonly used in experimental work is that invented by Daniell in 1836. This cell is shown in section in Fig. 87. It consists of a glass jar which contains a solution of copper sulphate in which a cylindrical sheet of copper is placed, and of a smaller jar within this, made of porous earthenware, and containing a solution of zinc sulphate into which dips a rod of zinc. When the cell is in action, copper is transferred to the copper cylinder and is there deposited, and there is no polarization.

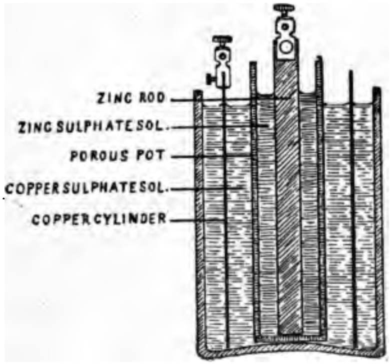

Fig. 87.

In setting up and using the cell observe the following directions:

(1) Polish the copper with sand-paper and wash till the surface is bright.

(2) Clean and amalgamate the zinc. First dip it into caustic soda solution, and wash with water. Then amalgamate. This is done by plunging it for a few seconds in dilute sulphuric acid, then rubbing it over with mercury.

(3) The porous cup should be cleaned and well soaked in water.

(4) Screw clamps should be cleaned with sand-paper.

(5) Fill the glass jar a little more than half full of copper sulphate solution.

(6) Fill the porous cup about half full of zinc sulphate solution (1 : 20 by weight).

(7) At the end of the experiment, take the battery and tray to the sink. Remove the zinc rod, and clean with cloth and water. Pour the zinc sulphate into a stock bottle or jar. If the zinc rod is blackened, throw the zinc sulphate away. Fill the porous cup with water and leave it in the sink. Remove the copper cylinder and wash it with water.

(8) The cell should not remain set up for more than about three hours at a time.

THE COMMUTATOR

In electrical measurements it is necessary to use a commutator for changing the direction of the current in some part of the circuit, or for switching from one circuit to another. A simple form of commutator, shown in Fig. 88, consists of a block of wood containing four holes filled with mercury, and a second block into which are inserted two U-shaped conductors of thick copper wire. The ends of the wires dip into the mercury cups.

The poles of the battery are joined to two diametrically opposite cups, say *a* and *b*, while the wires from the other part of the circuit, say from a galvanometer, are joined to the remaining two cups, *c* and *d*. If the block containing the conductors be placed upon the other block with the conductors dipping into the mercury cups, the current will flow through the galvanometer. To change the direction of the current through the galvanometer, the top block is lifted, turned through 90° in either direction about a vertical axis, and the conductors again dipped into the mercury cups.

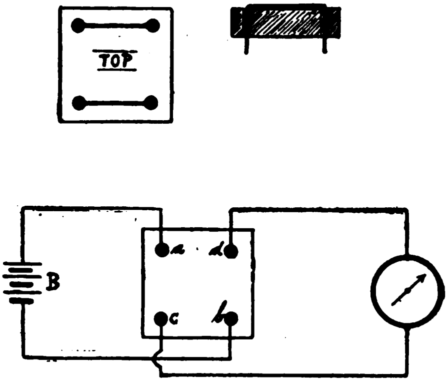

Fig. 88.

To break the current the block is lifted off. The commutator, therefore, may serve the purpose of a key.

Pohl's Commutator.—Instead of the commutator that has been described, that shown in Fig. 89 may be used. It consists of six mercury cups and six binding posts, the cups and posts being connected by conductors as shown. Each mercury cup is in connection with its nearest binding-post. A rocker, the two metallic ends of which are connected by an insulating handle S of glass or hard rubber, completes the circuit and changes the direction of the current when it is tilted over. Thus, let the terminals of the battery be connected with E and F, and the terminals of the galvanometer with A and B (Fig. 90). In the position shown, A is connected with F and B with E. But if the rocker is tilted over, A will be connected with E and B with F through the rocker conductors, and the current through the galvanometer will be reversed. To break the circuit the rocker is lifted off.

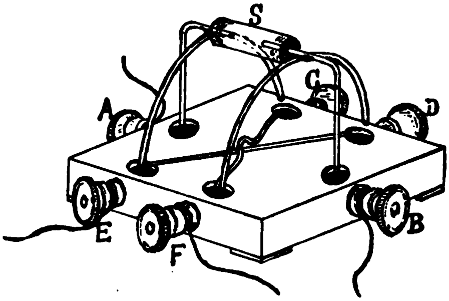

Fig. 89.

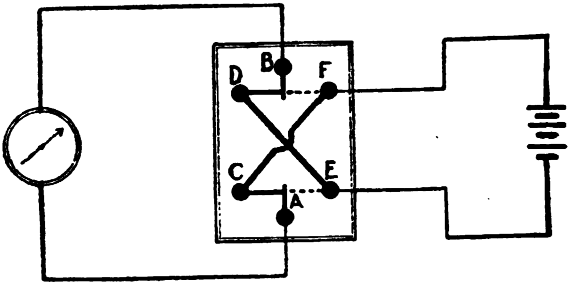

Fig. 90.

To determine the reduction factor (K) of a tangent galvanometer.

Apparatus: (1) A tangent galvanometer. (2) A gas voltameter. A 50 cc. burette can be used. (3) A battery of two or three Daniell cells. (4) A commutator. (5) A box of resistance coils. (6) A watch. (7) A Centigrade thermometer. (8) A barometer.

Errors: (1) Good electrical connections at all points are essential. The poles of the commutator must be clean.

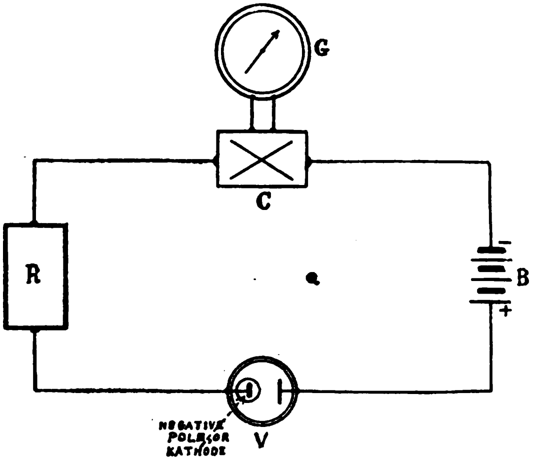

Fig. 91.

(2) A steady current must be maintained.

(3) Take great pains in determining the volume of gas collected.

Procedure: Connect the apparatus as in Fig. 91. G represents the galvanometer; V, the voltameter; B, the battery; C, the commutator; R, the box of resistances.

Fill the graduated tube carefully with water containing about 20 per cent. of sulphuric acid, and invert over the negative electrode. Hang the thermometer vertically alongside the tube. Allow the current to flow for a few minutes, adjusting in the meantime the number of turns of wire in the galvanometer coil, also varying the resistance in R if necessary, until the deflection of the needle is in the neighborhood of 45°. If by this time the apparatus appears to be in good working condition, break the circuit at the commutator. Note the position of the top of the column of liquid in the tube by means of the scale divisions etched on the glass. Take the reading at the under side of the meniscus, observing the usual precautions regarding parallax.

Close the circuit through the commutator, and note the exact second at which the current is made. Read the deflection of the needle at each end of the pointer, quickly reverse the current through the galvanometer, and read the deflection again. Allow the current to flow for at least 15 minutes, or until the tube contains a little less than its volume of gas. At regular intervals, say two minutes each, reverse the current through the galvanometer, and note each time the deflection. In each case read both ends of the pointer. Watch the needle carefully, and keep the current and deflection constant by varying the resistance in R. When a sufficient quantity of gas has been collected, break the circuit at the commutator, and note the exact time at which this is done. Note again the position of the top of the liquid in the tube. The temperature of the gas may be taken to be the same as that of the air. Read the thermometer, then read the barometer. Measure carefully the height in centimetres from the surface of the liquid in the vessel to the top of the liquid in the tube.

From the observations that have been taken, find the time in seconds during which the current decomposed the electrolyte into hydrogen and oxygen, the volume of hydrogen collected under the conditions of the experiment, and the mean deflection of the needle during the whole time. Look up the tangent of the angle of deflection in a table of tangents. Find the true pressure upon the gas in centimetres of mercury, then reduce the volume to N. P. T. Compute the current in amperes, and find the reduction factor of the galvanometer. Record as follows:

Voltameter reading at beginning,
Voltameter reading at end,
Volume of hydrogen gas,
Time of closing circuit,
Time of breaking circuit,
Time of flow of current (seconds),
Deflections,
Mean deflection,
Temperature,
Barometer reading,
Height of liquid in tube
  above liquid in vessel,
Current strength, amperes. Reduction factor,

Exercises: (1) Express the strength of the current in C. G. S. electromagnetic units.

(2) A current of 1 C. G. S. unit is passing through a wire bent into a circle of 10 centimetres radius. Find the force exerted on a unit magnetic pole at the centre of the circle.

(3) A certain dynamo used for electroplating is found to deposit 141.696 grammes of copper in one hour. What is the current furnished?

(4) A cell containing a solution of silver and a cell containing acidulated water are placed in a circuit and a current allowed to flow through them for ten minutes. If 0.012 gramme of hydrogen is evolved, what is the current strength, and how much silver was deposited?

To measure the strength of an electric current.

Apparatus: (1) A tangent galvanometer whose reduction factor is known. (2) A battery. (3) A commutator.

Errors: (Given in the preceding experiment.)

Procedure: Knowing that the strength of a current is proportional to the tangent of the angle of deflection, and having found the reduction factor of the galvanometer, the strength of a current flowing through a circuit, of which the galvanometer forms a part, can be measured.

Set up the apparatus as in the preceding experiment, omitting the voltameter and using a different battery if desired. A resistance may be introduced in the box of coils, but its value need not be known. When all contacts are good and the galvanometer properly adjusted, close the circuit at the commutator. Read the ends of the pointer, quickly reverse the current, and read again. Take the mean of the readings, and look up the tangent of the angle of deflection. Compute the current in amperes and also in absolute units.

Exercises: (1) Given .09 as the reduction factor of a galvanometer, find the current flowing when a deflection of 36° is obtained.

(2) Compare the currents which when flowing through a tangent galvanometer cause deflections of 20° and 45° respectively.

(3) If a certain galvanometer shows a deflection of 35° when a current of one ampere flows, what deflection will it show when a 1.5 ampere current flows?

THE D'ARSONVAL GALVANOMETER

In any galvanometer the two essential parts are a magnet and a coil of wire. Since the action between a magnet and a coil carrying a current is reciprocal, it is immaterial whether the coil or the magnet is free to move.

In the d'Arsonval galvanometer the coil of wire is movable, while the magnet is fixed (Fig. 92). A compound horse-shoe magnet of steel is supported on a stand. The coil is made up of a large number of turns of fine wire and swings in the strong field between the poles of the magnet. It is suspended by a fine wire or thin phosphor-bronze strip, while a straight wire or helix connects it to the stand from below. These wires also serve to lead the current to and from the coil. The magnetic field is concentrated on the coil by a fixed soft iron cylinder placed inside the coil. When a current flows through the coil, it tends to turn so that its plane, which normally coincides with that of the poles of the magnet, is at right angles to the lines of magnetic induction. The rotation of the coil is resisted by the torsion of the wire which suspends it,

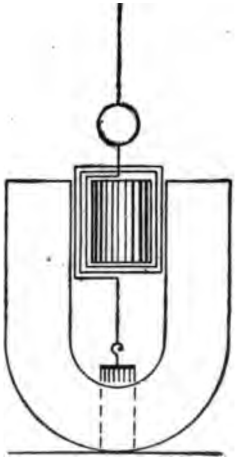

Fig. 92.

and the coil takes a position in which the couple of torsion of the wire balances that of the magnetic field. The great advantages of the d'Arsonval galvanometer are: (1) that it has a strong field of its own, hence is not affected by the earth's magnetism and by magnets or magnetic substances near it, (2) that it can be made so that the coil almost immediately assumes its position of rest.

At the upper end of the coil is attached a small mirror. A scale is placed in front of the mirror and the images of its graduations are observed by means of a telescope (Fig. 93). According to the law of reflection, the angle between the reflected ray and the perpendicular to the reflecting surface is equal to the angle between the incident ray and the perpendicular. On looking through the telescope at the mirror when in the position *a*M*b*, the division at O will be seen by direct reflection. If the mirror be turned through an angle *d* to the position *a'*M*b'*, the incident ray coming from a division at D makes an angle 2*d* with the reflected ray. Let *s* be the number of scale divisions in OD for a deflection *d*

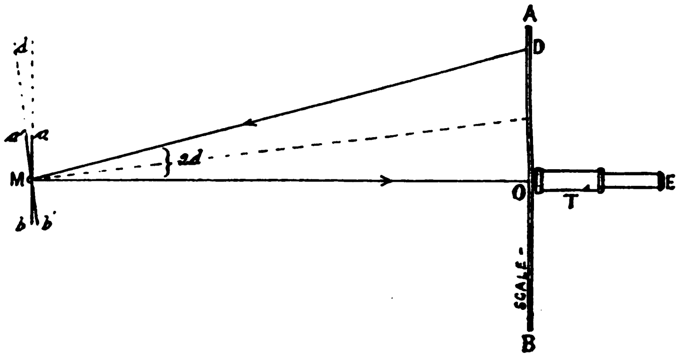

Fig. 93.

of the mirror, and R the distance from the mirror to the scale measured in units of the scale, then

$$\tan 2d = \frac{s}{R}.$$

If R is large in comparison with s, the error committed in taking the distances on the scale instead of the tangents will be inconsiderable; that is to say, the current strengths may be considered as directly proportional to the scale readings.

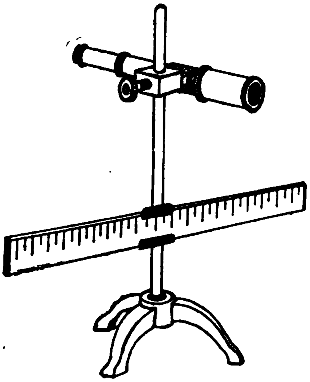

Fig. 94.

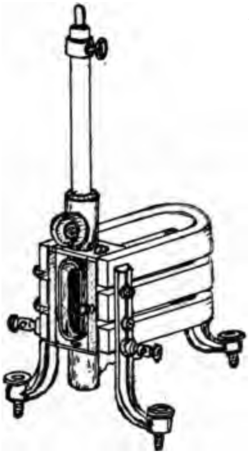

Fig. 95.

A galvanometer of this kind may be calibrated to indicate currents in amperes. A tangent galvanometer whose reduction factor K is known is joined in series in a circuit with the one to be calibrated. Different currents are passed through the instruments, the deflections are observed, and the current strengths are computed. A curve is then plotted connecting the deflections with the corresponding values of the currents.

To determine the relation between current and deflection in a d'Arsonval galvanometer.

Apparatus: (1) A d'Arsonval galvanometer (Fig. 95). (2) A tangent galvanometer whose reduction factor is known. (3) A reading telescope and scale. (4) A commutator. (5) A Daniell battery. (6) A box of resistance coils.

Errors: (1) All electrical connections must be good. (2) The two galvanometers should be placed at a considerable distance apart, else the magnet of the one may affect the field of the other.

Procedure: Connect the apparatus as shown (Fig. 96). D is the d'Arsonval galvanometer; T, the tangent galvanometer; B, the battery; C, the commutator; R, the box of resistance coils; A, the telescope and scale.

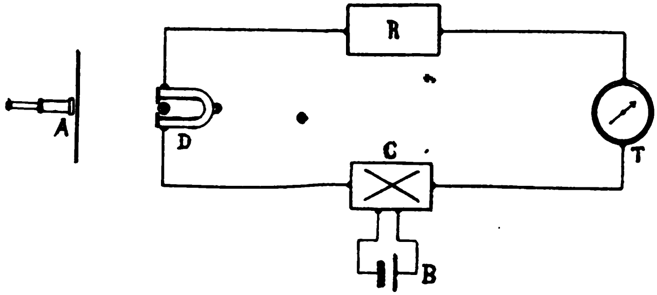

Fig. 96.

Adjust the telescope on the stand with its axis in line with the middle of the mirror. Clamp the millimetre scale to the front of the telescope support, so that it projects about the same distance on both sides. Then move the support and adjust the apparatus until the image of the middle division of the scale is distinctly focussed on the vertical cross-hair in the eye-piece. Close the circuit at the commutator, and make a preliminary test of the apparatus. Arrange the resistance in R so as to get a deflection of about 60° in the tangent galvanometer. If the deflection in the d'Arsonval galvanometer is too large, connect a wire between the terminals at the binding-posts of the instrument to direct a portion of the current from the coil (Fig. 97). The shorter and thicker the wire, the less the current that will flow through the galvanometer. The current flowing through the shunted galvanometer will always be a definite fraction of the total current flowing in the circuit; that is to say, the current through the galvanometer and the main current will increase and decrease proportionately. When the main current is doubled the current through the galvanometer is doubled, and son on. Hence it is possible to find by means of the following observations the general relation between the deflection in a d'Arsonval galvanometer and the current.

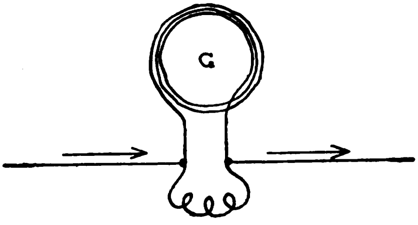

Fig. 97.

Break the circuit and determine the zero reading of each galvanometer. The zero of the d'Arsonval galvanometer should be noted from time to time. Close the circuit, read carefully the deflection in each galvanometer; reverse the current, and read the deflections again. Find the strength of the current, or simply take the tangent corresponding to the deflection in T. Continue these observations with gradually increasing resistances giving in succession deflections, say of about 55°, 50°, 45°, etc., down to 5°. Tabulate the numbers expressing the relative values of the currents, together with the galvanometer readings as follows:

Tangent Galvanometer.	**d'Arsonval Galvanometer.**	**Relative Currents.**

Plot a curve, using the d'Arsonval galvanometer deflections as ordinates and the corresponding currents as abscissae.

OHM'S LAW

The conception of a current of electricity involves the idea of a pressure needed to produce a flow. Take an analogy from hydrostatics. If two vessels containing water are connected by a pipe the flow will be from the vessel in which the water stands at the higher level into the vessel containing the water at the lower level. The strength of the current will depend on the pressure due to the difference in level. In the case of a Voltaic cell the current may be said to be due to an *electrical pressure;* and, to carry the analogy further, since that which moves matter is called force, that which causes the electric current is called electromotive force.

As in the case of the water flowing through the pipe the difference in pressure between any two points in the pipe can be measured by the difference in level between those points, so in the case of the electric current the electromotive force between any two points of a circuit is measured by the difference of pressure between those points. Imagine a wire of which one end A is kept at an electric pressure higher than that of the other end B (Fig. 98). Denote the difference of pressure, which

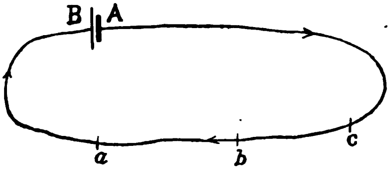

Fig. 98.

is a measure of the electromotive force between the ends, by E. In consequence of this difference of pressure there will be a flow of current from A to B, the strength of which will depend, among other things, upon the value of E. Double E and the flow is doubled, make E five times as great and the flow is increased in the same proportion, and so on. Thus, the strength of the current C is proportional to the difference of pressure E.

Somewhat as in the hydrostatic analogy the pipe will offer more or less resistance to the flow of water, according to its section, length, etc., so the wire will offer more or less *electrical resistance* according to its section, length, material, and physical condition. This resistance is a constant for one definite conductor and is denoted by R. R varies directly as the length of the conductor, because, for a given difference of pressure, the resistance to the flow in a conductor of given size and material is made twice as great by making the conductor twice as long. Also, if the cross-section of the wire be doubled, virtually making two equal conductors side by side, the flow of current will be doubled; in other words, the resistance of the wire is said to become one-half as great.

Consider a steady current C flowing in the conductor AB, and choose such portions of the conductor, as that between the points *c* and *b*, and that between the points *b* and *a*. The difference of pressure required to maintain the current from *c* to *a* will obviously be greater than that required to maintain the current from *c* to *b*; so that, if *ca* be twice the length of *cb*, the difference of pressure between *c* and *a* will be twice that between *c* and *b*. Also, since the resistance of a uniform conductor is proportional to its length, the resistance of *ca* will be twice that of *cb*. It follows, then, that the electrical pressure is proportional to the resistance, C being constant. That is to say, that portion of the total electrical pressure exerted in forcing a given current through any part of a conductor is proportional to the resistance of that part. Then

$$C = \frac{E}{R}.$$

This exact law for steady currents was discovered by Dr. G. S. Ohm, of Berlin, in 1827, and is known as Ohm's Law.

If several conductors, of resistances R', R'', R''', etc., are joined in series, and a current of strength C forced through,

$$C = \frac{E'}{R'} = \frac{E''}{R''} = \frac{E'''}{R'''} = \text{etc.},$$

E', E'', and E''' being the differences in pressure at the terminals of the different sections *ab, bc, cd,* etc. (Fig. 99).

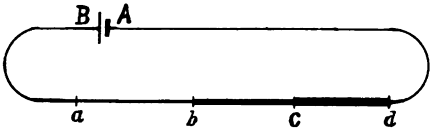

Fig. 99.

Therefore

$$C = \frac{E' + E'' + \ldots\ldots}{R' + R'' + \ldots\ldots} = \frac{E}{R},$$

where E is the total difference of pressure and R the total resistance.

To determine the relation between the electromotive force along a conductor and the resistance.

Apparatus: (1) A thin German-silver wire which is stretched tight along a board. A Wheatstone's wire-bridge will serve the purpose. (2) A contact key. (3) A d'Arsonval galvanometer with telescope and scale. (4) One or two Daniell cells. (5) A commutator. (6) A resistance box.

Errors: (1) There is a decrease in the electromotive force when the battery is allowed to run continuously, hence the current in the wire diminishes. (2) The resistance of the wire increases due to the heating effect of the current. (3) Make good electrical connections between the sliding contact and the wire.

Procedure: Connect the apparatus as shown (Fig. 100). *ab* is the German-silver wire; B, the battery; G, the galvanometer; R, the resistance box; C, the commutator.

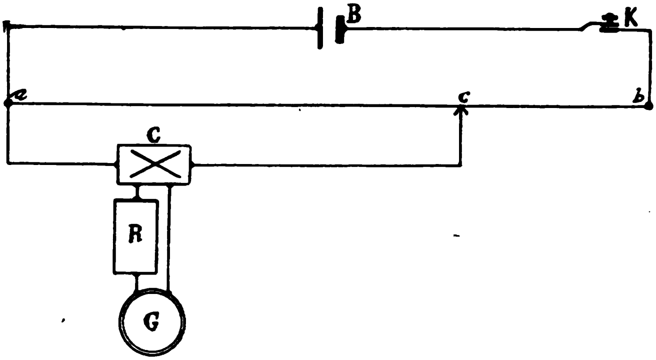

Fig. 100..

K is the key for closing the battery circuit and C the sliding contact key. The terminal of the galvanometer at *a* is fixed and should be at the end of the scale. When the sliding contact is applied to any point of the wire the circuit is shunted through the resistance R and the galvanometer. If the resistance of the galvanometer circuit is made very large, the current taken away from that in the wire will be too small to produce any sensible effect on the electrical pressure in the wire. The current through the galvanometer is proportional to the electromotive force between *a* and *c*.

Place the sliding contact key so that the entire length of the wire is included between the galvanometer terminals. Close the battery circuit and observe the deflection.

Adjust the resistance until the deflection is small in comparison with the distance from the mirror to the scale, say, about one-tenth. If the deflection is still too large when no more resistance is available, shunt the galvanometer.

Open both circuits. Place the sliding contact about 10 centimetres from *a*. Note on the scale the reading of the point where contact is made, and also note the exact position of the terminal at *a*. Determine the zero of the galvanometer. Close both keys and read the deflection; reverse the current through the galvanometer, and read again. Take the mean as the true deflection. When this is done open the circuits. Move the contact to a point 20 centimetres from *a*, and repeat the observations. Continue in this manner, placing the contact at distances of 10 centimetres along the wire until *b* is reached. Note the zero reading of the galvanometer two or three times during the experiment.

Reverse the poles of the battery, and repeat the observations, taking the same points in the opposite order. The two sets of results may be slightly different. The mean of the corresponding deflections should be taken.

Tabulate results as follows:

Length of Wire *ac*	Deflections		Mean D	$\frac{D}{ac}$
	I	II		

State briefly how the results go to prove that the electrical pressure along the wire is proportional to the resistance. How do the results verify Ohm's law? Plot a curve taking the lengths of wire as ordinates and the deflections as abscissae.

Exercises: (1) Why is a German-silver wire used in this experiment in preference to a wire of different material, for example, copper?

(2) Why is it necessary that the wire be uniform in cross-section and physical properties?

(3) A copper wire and a German-silver wire of the same size are joined in series in an electric circuit. It is found that there is the same difference of pressure between two points on the G. S. wire 7 centimetres apart as between two points on the copper wire 89.6 centimetres apart. Compare the resistances of G.S. and copper.

(4) The total resistance of a circuit is 18 ohms. What change in the strength of the current will be produced if two points of the circuit between which the resistance is 12 ohms are joined by a wire of 4 ohms resistance?

MEASUREMENT OF RESISTANCE

It has been shown that the electromotive force along a conductor is proportional to the resistance, the current being constant. Further, if different currents are made to pass through the same conductor by varying the difference of pressure between its terminals, the ratio of the current strength to the corresponding electromotive force is a constant. This constant is the *resistance* of the conductor. Thus, if an electromotive force E applied to a given conductor produce in it a current C, the resistance of that conductor is given by the ratio $\frac{E}{C}$. That is, if R denote the resistance of the conductor,

$$R = \frac{E}{C}.$$

Let E and C be expressed in absolute electromagnetic units, then R is also expressed in absolute units. A conductor possesses an absolute unit of resistance when a unit difference of pressure between its terminals causes a current of unit strength to pass through it. But this unit is inconveniently small for practical purposes. The practical unit of resistance is equal to 10^9 absolute units, and is called the *ohm*. The international ohm, adopted by a Congress of Electricians convened at Chicago in 1893, is represented by the resistance offered to a uniform electric current by a column of mercury at 0°C., 106.3 centimetres long, of uniform cross-section, and containing 14.4521 grammes.

In preparing practical standards of resistance, it is difficult to arrange columns of mercury in a form convenient for general use; hence the resistances used in practical measurements are generally sets of coils of insulated wire wound on bobbins, and their values so arranged that they may be conveniently used in different combinations. In making a coil, the wire is bent double at its middle point, and is wound as a double spiral, so as to have no self-induction. For convenience it is customary to place the coils within a box, thus constituting what is called a *resistance box*. The terminals of these coils are attached to thick brass plates fitted in rows on the top of the box (Fig. 101). Plugs of brass are made to fit snugly into grooves provided for them between the brass plates. When all the plugs are inserted, the current passes from plate to plate without going through the coils, since the plugs and plates offer practically no resistance. When any plug is taken out the current must pass through the coil bridging the space between the separated plates. Thus, any particular resistance can be introduced into a circuit by withdrawing the corresponding plug.

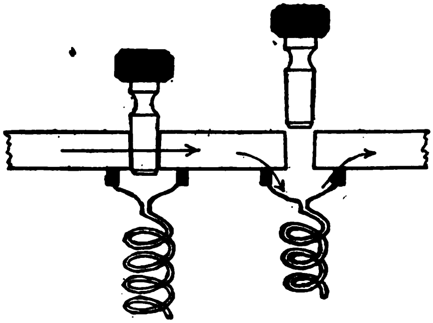

Fig. 101.

Some of the most necessary precautions to be taken in the use of a resistance box may be given as follows:

(1) Each plug is made to fit exactly its own opening between two adjacent plates. Therefore, the plugs should never be misplaced.

(2) The plugs and openings should be kept clean, otherwise an additional resistance is introduced.

(3) When inserting a plug apply a firm pressure together with a slightly twisting motion; but care should be taken not to insert it too rigidly.

(4) A current stronger than a few tenths of an ampere should never be passed through the coils.

RESISTANCE AND DIFFERENCE OF ELECTRICAL PRESSURE

It follows from Ohm's law, that if the same current passes through two conductors, the differences of pressure between their ends are to one another as their resistances. Let C be the current, E the difference of pressure between the ends of one conductor, and R its resistance; also let E′ and R′ be the corresponding quantities for the other conductor. Then, by Ohm's law,

$$R = \frac{E}{C} \text{ and } R' = \frac{E'}{C},$$

or

$$C = \frac{E}{R} \text{ and } C = \frac{E'}{R'}.$$

Whence

$$\frac{E}{R} = \frac{E'}{R'}$$

To measure a resistance by difference of pressure.

Apparatus: (1) A d'Arsonval galvanometer with telescope and scale. (2) An unknown resistance. (3) A resistance box. (4) A Daniell cell.

Errors: The same remarks apply here as in the three preceding experiments.

Procedure: Connect the apparatus as shown (Fig.

102), at first connecting the terminals of the galvanometer to the ends of the conductor X, whose resistance is to be found. Close the circuit at the commutator. If the deflection of the galvanometer is too great, reduce the current by shunting the battery, that is, connecting the poles by a short thick wire. Observe but do not record the deflection. Transfer the galvanometer terminals to the binding-posts of the resistance box. Adjust the resistance in the box so as to obtain a deflection about the same as that given when the terminals were applied to X, the unknown resistance. Read and record the deflection; reverse, and read again. Again apply the galvanometer terminals to X; read and record the deflection as before. Repeat the above experiment twice, varying the current somewhat each time by means of the shunt. Also slightly vary the resistance in R. Consider the results as follows:

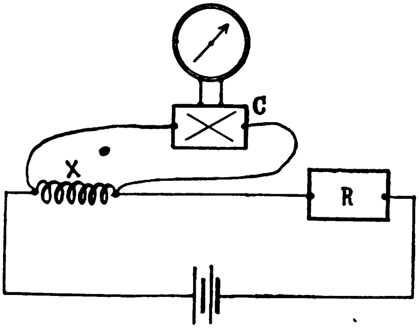

Fig. 102.

On what does the deflection of the galvanometer in each case depend?

State the relation between the resistance of a conductor and the difference of pressure between its ends

Express the ratio of the resistances in R and X in terms of the galvanometer deflections.

Then, in each case, knowing the value of the resistance in R and the galvanometer deflections, how may the value of X be found?

Record results as follows:

Galvanometer Deflections.		Resistance in Box R.	Ratio of Deflections.	Resistance of X.
R	X			

THE WHEATSTONE BRIDGE

By Ohm's law, the electromotive force or difference of pressure between any two points in a conductor carrying a current, is proportional to the resistance of the conductor between the two points. Let ABC and ADC be the two branches of a divided circuit (Fig. 103). If by means of a battery a current is made to flow from A to C, there is a continuous fall of pressure along each branch from the point at which the two parts of the current separate to the point at which they reunite. The pressure will vary along each branch from its value at A to its value at C. Let B be a point on ABC. Then

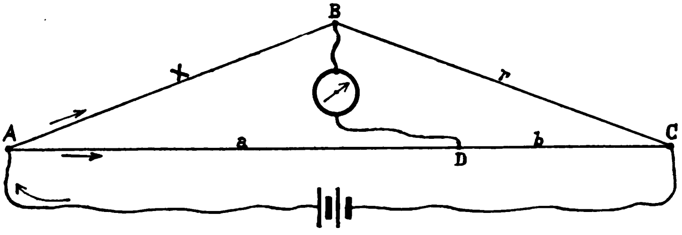

Fig. 103.

there must be some point on ADC which is at the same pressure as B. If two such points be connected by a wire, no current will flow, and if in this circuit a galvanometer be inserted, the needle will not be deflected, however large a current may be flowing through the divided circuit. Let D be the point at the same pressure as B. Denote the resistances in the parts of branch ABC by x and r, and the resistances in the parts of branch ADC by a and b. As no current can flow through the "bridge" BD, we have from Ohm's law:

$$\frac{\text{Difference of pressure between A and B}}{\text{Difference in pressure between B and C}} = \frac{x}{r}, \qquad (1)$$

and

$$\frac{\text{Difference of pressure between A and D}}{\text{Difference of pressure between D and C}} = \frac{a}{b}. \qquad (2)$$

Since B and D are at the same pressure, the difference of pressure between A and B is equal to the difference of pressure between A and D, and the difference of pressure between B and C is equal to that between D and C. The first members of equations (1) and (2) are therefore equal. Hence,

$$\frac{x}{r} = \frac{a}{b}, \text{ or } x = \frac{a}{b} r. \qquad (3)$$

If three of the four resistances are known, the fourth can be computed. If the branch ABC be a uniform straight wire, the resistances of the parts are proportional to their lengths. Therefore we have

$$x = \left(\frac{\text{length AD}}{\text{length DC}}\right) r. \qquad (4)$$

To measure a resistance by the Wheatstone bridge.

Apparatus: (1) A Daniell cell. (2) A Wheatstone bridge. (3) A sensitive galvanometer. (4) A resistance box. (5) Two coils of wire whose resistances are required.

Errors: (1) Good electrical connections at all junctions are necessary. (2) The coil whose resistance is to be measured should be so placed that the current flowing through it will not affect the galvanometer directly.

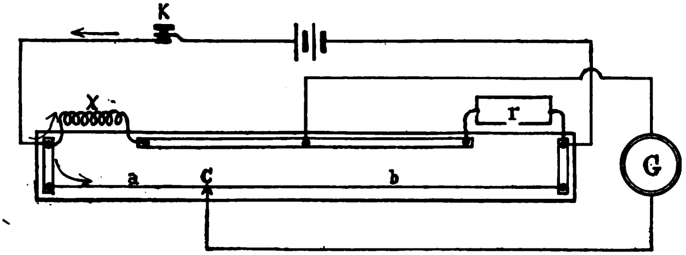

Fig. 104.

Procedure: Connect the apparatus as shown (Fig. 104). *x* is the unknown resistance; *r*, the known resistance given in the resistance box; B, the cell; G, the galvanometer; K, the key in the battery circuit; C, the sliding contact key. Make the junctions at *x* and *r* by means of short, thick copper wire, whose resistance may be neglected. It is now required to find such a position for C on the stretched wire that when good contact is made and the battery key K is closed, no current shall flow through the galvanometer. Make contact with C at about the middle of the wire. If the needle moves, note the directon of its motion. Move the sliding contact to a new position, and again note the effect upon the galvanometer. If the two trials give opposite deflections, the equal pressure point must be somewhere between the two positions, Move C along the wire until the deflection is reduced to zero. On making or breaking contact at C, no effect should be observed when contact is made for a second or two. At each trial the contact at C should always be made after the battery key is closed; that is, close the battery key first and open it last. It is best to introduce such resistance at r that the equal pressure point will not be far from the middle of the wire. When a point has been found so that no current flows through the galvanometer, record the length of a and of b; also record the resistance in r. Compute the resistance x. In the same manner determine the resistance of the other coil. Connect the two coils in series, and determine the resistance. Compare this result with the sum of the separate resistances as determined. Connect the two coils in parallel, and determine the resistance. Compare this result with the calculated resistance of the two in parallel. Tabulate as follows:

Coils.	a	b	r		**Computed Resistances (Series and Parallel)**
1st Coil.					
2nd Coil.					
Coils in series.					
Coils in parallel.					

Exercises: (1) Is it strictly necessary to use a battery of constant current in this experiment? Why?

(2) Prove that battery and galvanometer may be interchanged without affecting the balance of the bridge.

(3) When r equalled 2.8 ohms, C was found at the 40 centimetre mark. What was the value of x, and where must C be placed if r is changed to 3 ohms?

(4) What "shunt" must be put in parallel with a coil of wire which has a resistance 1000, in order that only one-tenth of the current may flow through the coil?

RESISTANCE BY SUBSTITUTION

By Ohm's law, if the electromotive force in a circuit remains constant, the resistance will be inversely proportional to the current. If the electromotive forces and currents in two circuits are the same, the resistances of the circuits are also the same. Or, in any circuit with a given electromotive force and current, if a portion of the resistance be removed and replaced by another resistance, and if the electromotive force and current still remain unchanged, the replacing resistance is equal to that which has been replaced.

To measure a resistance by substitution.

Apparatus: (1) A sensitive galvanometer. (2) A Daniell cell. (3) A resistance box. (4) A resistance to be measured.

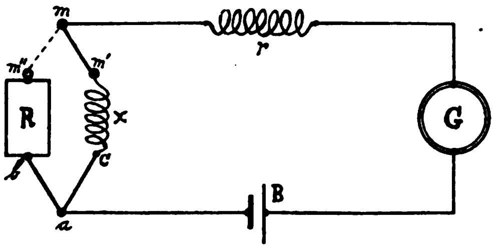

Fig. 105.

Errors: This method assumes that the electromotive force remains constant during the determination. Hence the circuit should be left closed only while readings are being taken.

Procedure: Connect the apparatus as shown (Fig. 103). G is the galvanometer; B, the battery; R, the resistance box; X, the unknown resistance. Mercury cups may be used at m, m' m''. ab and ac are equal lengths of thick copper wire. Note the reading of the galvanometer. Throw the unknown resistance X into the circuit by dipping the ends of a short copper wire into the mercury cups m and m'. If the galvanometer deflection is too large, a resistance r may be put into the main circuit sufficient to bring the deflection to the desired amount. Read the deflection of the galvanometer. Throw R in circuit by putting the ends of the wire in m and m''. Adjust the resistance in R until the deflection is exactly the same as before. Note the resistance in R. Then, if the electromotive force has remained constant, it is obvious by Ohm's law that

$$R = X.$$

Repeat the measurement, varying the current in the main circuit, so as to obtain a somewhat different deflection of the galvanometer.

RESISTANCE AND ELECTROMOTIVE FORCE

Because a voltaic cell possesses the property of maintaining its electrodes at a difference of pressure, it is said to be the seat of an electromotive force. The electromotive force is directly proportional to, and can be measured by the greatest difference of pressure that is found to exist between its terminals when the circuit is open. It depends on the nature of the materials composing the cell; that is, on the nature of the liquid and of the plates, but not on the dimensions of the plates or their positions in the liquid.

The unit of electromotive force in the C. G. S. system is inconveniently small for practical purposes. The unit generally adopted is the *volt.* This unit is equal to 10^8 units in the C. G. S. system, and may be defined as the difference of pressure that must be maintained between the terminals of a conductor of one ohm resistance in order to produce a current of one ampere.

Each voltaic cell offers a resistance to the passage of the current which it produces. The value of this resistance depends to some extent on the nature of the plates of the cell, on the amount of their surfaces submerged, and on their distance apart. It may also depend slightly on the composition of the battery solutions.

The current which a cell can send through a circuit depends on the electromotive force producing the current, and on the resistance which the circuit offers to the flow of the current. This resistance is the sum of two, that offered by the cell itself, and that offered by the circuit outside the cell.

By Ohm's law, the current generated is equal to the electromotive force divided by the resistance of the circuit. Let E denote the electromotive force of the cell, R the resistance of the outside circuit, B the battery resistance, and C the current. Then

$$C = \frac{E}{B+R}.$$

The total resistance of a circuit can be found by a method which involves a direct application of Ohm's law. Take a circuit including a battery cell, a tangent galvanometer, a variable known resistance, and connecting wires.

Let E = Electromotive force of the cell.
B = Resistance of cell, galvanometer and connecting wires.
R and R′ = Known resistances in the circuit.
d and *d′* = Angles of deflection.

Then, from Ohm's law, the current C passing through the galvanometer when R is in the circuit will be,

$$C = \frac{E}{B+R}. \qquad (1)$$

But

$$C = K \tan d, \qquad (2)$$

by the theory of the tangent galvanometer.

Hence

$$\frac{E}{B+R} = K \tan d. \qquad (3)$$

Let R be changed to R′ causing *d* to be changed to *d′*

then

$$\frac{E}{B+R'} = K \tan d'. \qquad (4)$$

Dividing (3) by (4),

$$\frac{B+R'}{B+R} = \frac{\tan d}{\tan d'}, \qquad (5)$$

and

$$B = \frac{R \tan d - R' \tan d'}{\tan d' - \tan d}. \qquad (6)$$

By making tan $d' = \frac{1}{2}$ tan d, the formula becomes

$$B = R' - 2R.$$

As the resistance of the galvanometer and connecting wires can be made small compared to that of the cell, the value of B can be taken as approximately the resistance of the cell.

Also, by applying Ohm's law to a complete circuit, the electromotive force of a cell can be found.

Let E = Electromotive force of the cell.
R = Resistance of cell, galvanometer, coils. and connecting wires.
r = An added known resistance.
d and d' = Angles of deflection.

Then the current C passing through the galvanometer, when R is the total resistance in the circuit, will be

$$C = \frac{E}{R} = K \tan d. \qquad (1)$$

Let a known resistance r be added, causing d to be changed to d', then the current C' will be

$$C' = \frac{E}{R + r} = K \tan d'. \qquad (2)$$

Dividing (1) by (2),

$$\frac{C}{C'} = \frac{R + r}{R}, \qquad (3)$$

and

$$R = \frac{C'r}{C - C'}. \qquad (4)$$

The values for C, C′, R, and r being known, the electromotive force can be found from

$$E = CR,$$

or

$$E = C'(R + r).$$

To measure the resistance of a circuit.

Apparatus: (1) A tangent galvanometer of low resistance. (2) Two Daniell cells. (3) A commutator. (4) A resistance box.

Errors: (1) The method of this experiment assumes that the electromotive force of the battery remains constant when the current is changed. The circuit should be closed only long enough at a time to take a reading. (2) The deflections of the galvanometer must be neither very large nor very small.

Procedure: Connect a single cell in circuit with the tangent galvanometer and the resistance box (Fig. 106). The galvanometer should be reached through the commutator, in order that the deflections may be obtained to the right and to the left.

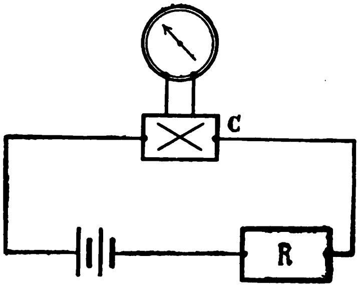

Fig. 106.

Make the resistance R in the resistance box such that the deflection *d* shall be somewhat greater than 45°. Determine the mean deflection. Change the resistance in the box so that the deflection *d'* shall be about half *d*. Again take readings and determine the deflection. Find the tangents of the angles of deflection. Compute the resistance of the circuit. The result may be taken as practically the resistance of the cell, providing the resistance of the galvanometer and connecting wires has not been made large.

Repeat the above observations with the other cell. Next connect the two cells in series, and determine the resistance of the battery. Then connect the two cells in parallel and determine the resistance.

Record the observations as follows:
Galvanometer No.,

Cells.	Box Readings.		Defl'ct'ns		Tangents		Resistances.
	R	R'	*d*	*d'*	*d*	*d'*	B
1st Cell							
2nd Cell							
In Series							
In Parallel							

What appears to be the effect upon the resistance of the battery (*a*) when the cells are joined in series, (*b*) when they are joined in parallel?

To measure the electromotive force of a battery.

Apparatus: (1) A tangent galvanometer whose reduction factor is known. (2) Two Daniell cells. (3) A resistance box. (4) A commutator.

Errors: Given in preceding experiments.

Procedure: Join the cell in series with the resistence box and tangent galvanometer, the latter being reached by the commutator. Introduce enough resistance to produce a deflection of about 50°. Determine the deflection and calculate the current C in amperes. Call the total resistance of the circuit R. Add enough resistance *r* to make the deflection about half what it was before. Record the value of the resistance added. Determine again the deflection and calculate the current C′. From the values of C, C′ and *r*, compute the resistance of the circuit when the current C was passing. Then, applying Ohm's law further, compute the electromotive force.

Repeat the above observations with the other cell. Next connect the two cells in series, and determine the electromotive force of the battery and the current in the circuit. Then connect the two cells in parallel and determine the current and the electromotive force. Record the resistances of the batteries as determined in the preceding experiment.

Record the observations as follows:
Galvanometer No.,

Cells.	Resistance Added. (*r*)	Circuit Resistances	Currents		Electromotive Force.
			C	C′	
1st Cell 2nd Cell Both in Series Both in Parallel					

What is the effect upon the electromotive force of the battery (*a*) when the cells are joined in series, (*b*) when they are joined in parallel?

Compare the values of the currents as found by experiment with the values of the same currents found by calculation.

Which of the two ways of joining cells will give the greater current, (*a*) when the external resistance is large, (*b*) when the external resistance is small? Illustrate by means of problems

Exercises: (1) If a cell have an electromotive force of two volts, and a resistance of one ohm, what current will it give through a wire so short and thick that its resistance is too small to be considered?

(2) A battery of 5 cells, joined in parallel, with electromotive force of 1.1 volts each and a resistance of 3 ohms each, is in closed circuit with an outside connecting wire whose resistance is 10 ohms; required the strength of current in the circuit.

(3) A telegraph sounder requires 0.1 of an ampere to work it properly; if a battery of 4 cells, each with 1.5 volts, be in series in the circuit, what must be the resistance of the circuit that it may be worked?

APPENDIX

TABLE FOR THE PHYSICAL LABORATORY

The laboratory table should have a heavy top and should be provided with an adjustable cross-bar which can be fixed at different heights. The dimensions given below are for a table intended to accommodate four pupils working in pairs. Fig. 107 shows the front and end elevations, and also the plan of the top.

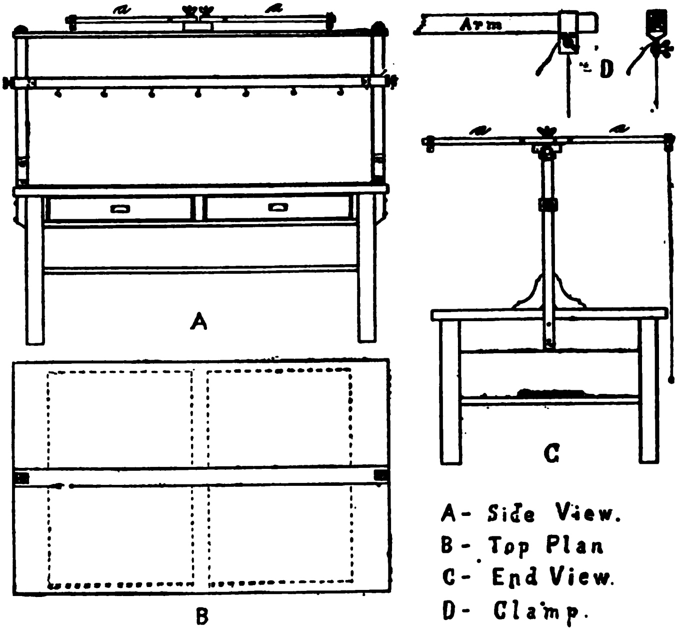

Fig. 107.

Top surface....	70″×48″	
Top above floor	34″	
Floor to top cross-bar	82″	
Lower platform above floor.	12″	
Width of lower platform.. .	15″	
Table legs	3½″×3½″	section.
tapering to	2½″×2½″	"
End posts.....	2½″×2″	"
Adjustable cross-bar	2½″×2″	"
Top cross-bar....	2″×1¾″	"

The top of the table should project beyond the frame 3 inches all around, so that apparatus can be clamped down. There are four drawers, each 6 inches deep. The adjustable cross-bar can be secured in the position desired by the device shown in A. If the metal collars and thumb-screws for the two end clamps cannot be obtained, the two end posts can be made 2½×2½ inches in cross-section and provided with 1 inch slots, in which the ends of the cross-bar are made to slip easily. About 12 small holes can be bored through the posts, permitting the insertion of loose pegs for supporting the ends of the bar, On the fixed cross-bar two hinged arms (*a* and *b* in C) may be provided and so constructed as to be clamped at any position by thumb-screws.

These arms serve to support pendulums, etc. A number of hooks fastened to the adjustable bar are also provided for similar purposes as shown. The clamp shown at D is made of stiff brass ¾ inch wide and bent into a form which will allow it to slip over the arm. By means of a thumb-nut the clamp can be tightened as shown in the cut. The platform under the table may be used for the reception of batteries, etc. For the purpose of connecting batteries, binding posts may be fixed to the table and connected with wires from below. It is intended that the gas pipe, provided with a two-way nozzle for the attachment of Bunsen burners, shall be let down from the ceiling of the laboratory to a point not far above the cross-bar. No iron should be used in the construction of the table. If screws must be used, they had better be of brass.

This table will not be found too large for two pupils, yet it may be necessary in some laboratories to use a table of smaller size. The top can be made 5 feet long by 3 feet wide, and the other dimensions reduced where necessary. The construction can be further simplified and cheapened by omitting the hinged arms and by having only one drawer on each side. The thin strip, shown in B, across the top surface can also be omitted.

APPARATUS AND MATERIALS

Many teachers may desire to improvise apparatus, provided time and experience will permit, but material furnished by the regular dealers will be found much more satisfactory and durable, while the prices asked by responsible dealers will not be found exorbitant.

MATERIALS FOR GENERAL USE

Acids (sulphuric, nitric, hydrochloric), alcohol, asbestos.

Balls (hardwood, lead), batteries (various kinds), battery connections, battery parts, beakers (copper and glass), bellows (foot), beam compass, binding posts, bits, bit stock, blast lamp, blow-pipe, boards, bottles, boxes, bristles (hog's), brushes (camel's hair), Bunsen burners.

Calcium chloride, candles (paraffine), cardboard, cat skin, caustic potash, chisel, clamps (various kinds), clamp stands, clamps (joiner's), compasses, commutators, contact keys, copper sulphate, cork, cork dust, cork borers, cross-section paper.

Distillation apparatus, dividers, drying-paper.

Emery-paper, ether.

Files (triangular and round), flasks (various sizes), funnels.

Glass tubing, glass plates, glass cutter, glass rods, gold leaf, graduates (metric).

Hammer.

Lamps (kerosene, alcohol), lamp chimneys, lead (sheet), leather, levels.

Magnetic needles, mercury, mercury trough, metre sticks, metric rules, mirror glass.

Needles, nails.

Oil, oilstone.

Paraffine, pins, paper scales.

Resistances (variable), ring-stands, rods (metal and wood), rosin, rubber tubing (common and pure), rubber dam, rubber cement, rubber stoppers.

Sand-paper, sealing-wax, screw-driver, shears, sieve (fine wire), soldering materials, spools, stands (wooden), string (linen and cotton).

Test-tubes (large size), thread (linen and silk), tinfoil, turpentine, tumblers.

Vise, violin bows.

Water bath, weights of higher denominations, as 1,000 grammes to 100 grammes, wire (copper, brass, iron, German silver, platinum), wire cutters, wrench.

Zinc, zinc sulphate.

APPARATUS FOR THE EXPERIMENTS

The experiments included in this book need not necessarily be performed exactly in the order in which they are given, and by judicious planning, a considerable saving in apparatus may be effected. As a rule, there should not be more than two different experiments going at the same time in a laboratory section. In some cases it is necessary that all pupils be engaged on the same experiment at the same time.

Pupils must be kept even in their laboratory work; there must be no lagging behind, and to that end, some pupils may need to work extra time in order to keep up with the class. In no case should pupils be allowed to leave an experiment and proceed to another, until their work has been found satisfactory. Extra study or investigation can be assigned those who may occasionally finish an experiment ahead of time. Experience has shown that it is a good plan to have pupils work in pairs. Pupils are held responsible for the apparatus they are using, and at the end of a laboratory period all pieces of apparatus must be left in the condition in which they were found. It will be necessary for practical reasons to have many of the pieces of apparatus numbered.

The number of pupils in a laboratory section is a matter that will depend upon the general school program as well as upon the laboratory accommodations, etc., In the following list of apparatus let it be assumed that there are sixteen pupils in a laboratory section—about as large a number as one teacher can conveniently handle together. The figures in parenthesis indicate how many of each piece of apparatus are needed. Where only one is required, no number is given. In all cases in which (8) occurs, it is intended that all pupils in the section shall be at work upon the same experiment at the same time; where (4) occurs, two experiments are to be performed at the same time, pupils exchanging places or apparatus when an experiment is completed. Where only one of a given kind of apparatus is indicated, it is intended that only one will be needed for the entire section. If the laboratory sections are to be larger or smaller than that assumed, the above plan of selecting material can still be carried out.

General Measurements.

Columbia steel rule (8), 6 inches long, graduated in millimetres on one side and hundredths of an inch on the other.

Vernier sliding caliper (4), graduated arm 10 centimetres long, graduated to read one-tenth of a millimetre.

Metric micrometer caliper (4), measuring to 0.01 millimetre.

Metal cylinder (8), of brass; must be accurately cut, 7 to 8 centimetres long, 4 to 6 millimetres in diameter, sharp edges, ends parallel and perpendicular to the length of the cylinder. It is advisable not to have either dimension an exact number of millimetres. The number of each cylinder may be marked on the end by means of a die.

Beam balance (4). The balance should be sensitive to one milligramme and mounted in a case with sliding door.

Set of weights (4), capacity 50 grammes to 0.01 gramme. The weights must be as accurate as the sensibility of the balance will warrant.

Pendulum ball (4). A lead ball with a wire loop for suspension will answer the purpose.

Support for pendulum (2). Use either the clamp shown in Fig. 14 or the device shown in D, Fig. 107.

Burette (4). A Mohr's burette, 25 or 50 cc. capacity, provided with spring pinchcock, is recommended, graduated to 0.1 cc.

Mechanics.

Micrometer screw and stand (4). A micrometer consisting of a large dial graduated in 100 parts and attached to a screw having a thread one millimetre in pitch is the most convenient form for this work. The support for the screw must be rigid so as not to allow any change in the relation of the parts during the course of the exercise.

Pine rod (8). These rods must be selected with great care and made of the best quality of stock. Dimensions must be regular. It is advisable to have the rods of two sizes, the dimensions bearing a simple ratio.

Pair of fulcrums (4), each provided with a knife edge.

Pan for suspending weights (4), provided with contact piece and binding post.

Jolly's balance (4). An extra spring should be furnished with each balance. A mirror scale, leveling screws, and adjustable support are essential.

Apparatus for demonstrating the laws of falling bodies, as follows: Tuning fork of 256 vibrations, adjustable support and slide for glass plate as shown in cut. Spring dividers are convenient in this exercise for taking measurements. The instructor can perform the experiment of dropping the smoked glass plates for the entire section. Pupils make the measurements as directed. Extra glass plates should be provided.

Impact apparatus (2), consisting of a wooden frame with cross-pieces, two platforms and stout threads for suspension.

Set of heavy weights (2), metric, 5000 to 500 grammes.

Frame for parallelogram of forces (4). Iron or wood clamps should be provided for clamping this frame to the table.

Spring balance (12). A spring balance similar to that shown in Fig. 23 should be used. The scale can be calibrated so as to give readings in grammes.

Apparatus for demonstrating the law of moments (4). The disc should be made so that it will not warp and should move in the bearings with as little friction as possible.

Apparatus for specific gravity of liquids (4). Use 15 per cent. solution of copper sulphate in distilled water.

Barometer. A barometer provided with a 0 point adjustment should always be used. If the regular Fortin instrument cannot be obtained, a good substitute can be had for $5 or $6. Schools have been giving insufficient attention to the use of the barometer in experimental work. The adjusting of the 0 point should be thoroughly understood by every pupil.

Apparatus for Boyle's law (4). A very satisfactory form of apparatus for this work is that shown in Fig. 41. This device is now on the market and has been carefully worked out.

Sound.

Resonance apparatus (4). A cork mallet is convenient for vibrating the tuning fork. It can be made by attaching a large cork stopper to a stiff brass wire.

Apparatus for finding velocity of sound in metals. One apparatus might here be made to accommodate the entire section, the instructor attending to the manipulation, the measurements being made in turn by the pupils. Cork dust is found superior to the lycopodium powder commonly used.

Heat.

Thermometer (8). A paper scale thermometer having a range of from—10° to 110° Centigrade is the most convenient form of instrument. Thermometers of both Fahrenheit and Centigrade scales can now be obtained at a low price and, when required, records can be taken in both systems. Thus the student will unconsciously obtain a working knowledge of both methods of calibration.

Boiler and steam cylinder (4). The low form of steam generator which has a screw cap and water gauge is preferable. The water gauge prevents burning out the boiler, the screw cap makes it possible to close the generator steam tight and to increase the internal pressure to any desired amount.

Linear expansion apparatus (4). If the micrometer screw form of linear expansion apparatus be used, great care must be exercised to keep the electrical contacts clean.

Calorimeter (8). This should be made of thin spring copper carefully nickeled.

National trip scale (2). This form of balance is the one largely used for general measurements in the study of physics.

Set of metric weights (2), 500 gr. to 1 gr. in block.

In specific heat determinations, use metal borings of copper or iron, lead shot or copper shot.

Light.

Optical bench with Bunsen photometer (4). A photometer in which the two sources of light and the mirrors are enclosed in a light-proof box will be found convenient.

Mirrors. Liberal provision should be made for the providing of mirrors so that each student may work independently. Concave and convex cylindrical and spherical mirrors now obtained at a low price are highly satisfactory.

Plate for refraction of light (8), consists of a piece of heavy plate glass with edges finished perpendicular to the plane.

Lenses. A good supply of lenses of different kinds, with lens holders, should be provided.

Small kerosene lamps will be found to serve most purposes better than candles.

Electricity and Magnetism.

Gold-leaf electroscope (8).

Pair of bar magnets (4). Magnets about 6 inches long provided with keepers are desirable.

Pocket compass (8). A compass 25 millimetres in diameter will be found of convenient size.

Blue print paper, of good quality.

Tangent galvanometer (4). This should have a large dial, short needle, and long, light pointer. A galvanometer so constructed that either pivotal bearing needle or a fibre suspended needle may be used will be found desirable.

Scales having both degrees and tangents marked off are now made and are convenient for use with the tangent galvanometer.

Electrolysis apparatus. This is to be used as a hydrogen voltameter in determining the reduction factor of the tangent galvanometer. All galvanometers may be joined in series with the voltameter, battery, etc., and the value of K for each instrument determined.

Daniell cell (8). This should be a large sized cell, and if properly set up, will have a constant electromotive force.

d'Arsonval galvanometer (4). An instrument suitable for the experiments described in this book and sufficiently accurate has been difficult to obtain at a price that schools could afford. The instrument shown in Fig. 95 has been thoroughly tested and found to be very satisfactory.

Reading telescope (4), with cross hairs, mounted on stand.

High resistance (4), 500 to 1000 ohms.

Resistance box (4). Carefully constructed boxes of plug form should be used.

Wheatstone slide wire bridge (4). This should be provided with a millimetre scale and either manganin or platinoid wire used in the construction.

Set of resistance spools (4). Spools wound with different lengths, sizes, and kinds of wire for determining laws of relative resistance, are now furnished, 8 in one set, mounted on a single base.

Pohl's commutator is recommended for all experiments requiring readings by direct and reverse currents.

Good storage batteries will be found useful in some experiments.

TABLES

I

Mensuration.

$\pi = 3.14159.$ $\pi^2 = 9.8696.$ $\frac{1}{\pi} = 0.3183.$ $\sqrt{\pi} = 1.7724.$

Circumference of a circle (radius r) $= 2\pi r.$
Area of a circle $= \pi r^2.$
Surface of a sphere $= 4\pi r^2.$
Volume of a sphere $= \frac{4}{3}\pi r^3.$
Surface of a right cylinder (radius r, altitude h) $= 2\pi r(r + h).$
Volume of a right cylinder $= \pi r^2 h.$

II

Metric Equivalents.

1 metre = 39.37 inches.
1 inch = 2.54 centimetres.
1 cubic inch = 16.387 cubic centimetres.
1 litre = 1.0567 quart = 0.2641 gallon.
1 gallon U. S. = 3.7854 litres = 231 cubic inches.
1 " " " = 8.3389 pounds water at 15° C.
1 kilogramme = 2.2046 pounds.
1 pound = 453.603 grammes.
1 ounce (Av.) = 28.3498 grammes = 437.5 grains.
1 ounce (Ap.) = 31.1034 grammes = 480 grains.
1 cubic foot = 7.48 gallons = 28.315 litres.
1 " " = 62.42 pounds water at 15°C.
1 fluid ounce = 29.5737 grammes water at 15°C.

III

Densities.

Solids.

The following numbers are to be regarded only as approximations, as the density varies with the temperature and with the specimen.

Aluminium	2.580	Iron, wrought	7.800
Antimony, cast	6.720	Iron, steel	7.790
Beeswax	0.964	Lead	11.350
Bismuth	9.800	Magnesium	1.750
Brass, cast	8.3 +	Nickel	8.900
Brass, sheet	8.440	Platinum	21.500
Bronze	8.700	Quartz	2.650
Copper, cast	8.830	Silver	10.400
Copper, sheet	8.900	Tin	7.290
Cork	0.240	Wood, cedar	0.554
Cobalt	8.800	" cherry	0.710
Diamond	3.530	" ebony	1.200
German silver	8.620	" maple	0.755
Glass, crown	2.520	" oak	0.779
Glass, flint	3.0 to 3.700	" pine	0.500
Gold	19.300	" walnut	0.680
Ice	0.917	Zinc, cast	7.100
Iron, cast	7.10 to 7.600		

IV

Densities.

Liquids at 15° C.

Acetic acid	1.053	Ether	0.720
Alcohol, ethyl	0.793	Glycerine	1.260
Benzine	0.890	Mercury, at 0°C	13.596
Carbon disulphide	1.270	Turpentine	0.872
Chloroform	1.499	Water	0.999

V

Densities.

Gases at 0° C. and 76 centimetres of Mercury Pressure

Air, dry	0.001293	Hydrogen	0.0000895
Ammonia	0.000770	Nitrogen	0.001257
Carbon dioxide	0.001974	Oxygen	0.001430
Chlorine	0.003133		

VI

Density of Water at Different Temperatures.

Degrees		Degrees	
0 C.	0.999878	16 C.	0.999004
1	0.999933	17	0.998839
2	0.999972	18	0.998663
3	0.999993	19	0.998475
4	1.000000	20	0.998272
5	0.999992	21	0.998065
6	0.999969	22	0.997849
7	0.999933	23	0.997623
8	0.999882	24	0.997386
9	0.999819	25	0.997140
10	0.999739	26	0.99686
11	0.999650	27	0.99659
12	0.999544	28	0.99632
13	0.999430	29	0.99600
14	0.999297	30	0.99577
15	0.999154	31	0.99547

VII

Density of Aqueous Solutions at 15° C. referred to Water at 4° C.

The percentage signifies the weight of the substance contained in 100 parts by weight of the solution. The salts are anhydrous.

%	Alcohol.	NH_4Cl.	$CaCl_2$.	$CuSO_4$.	KI.	KNO_3	NaCl.	$ZnSO_4$
5	0.9904	1.1049	1.042	1.050	1.037	1.031	1.035	1.052
10	0.9831	1.0299	1,086	1.103	1.077	1.064	1.072	1.108
15	0.9769	1.0443	1.130	1.161	1.119	1.099	1.110	1.168
20	0.9708	1.0584	1.811	1.225	1.165	1.135	1.150	1.236
25	0,9644	1.0721	1.232	....	1.217	....	1.191	1.307
30	0.9569	....	1.286	....	1.270	....	....	1.382
35	0.9485	....	1.343	....	1.330	....	....	....
40	0.9390	....	1.402	....	1.395	....	....	....
45	0.9287	....	....	....	1.468	....	....	....
50	0.9179	....	....	....	1.545	....	....	....

VIII

Velocity of Sound.

	Degrees	Cm. per Sec.		Degrees	Cm. per Sec.
Air	0 C.	33,250	Water	4 C.	140,000
Hydrogen	0	128,600	Brass	..	350,000
Illuminating gas	0	49,040	Copper	20	356,000
Oxygen	0	31,720	Glass	..	506,000
Alcohol (absolute)	8.4	126,400	Iron	..	509,300
Petroleum	7.4	139,500	Paraffine	16	130,400

IX

Centigrade and Fahrenheit Degrees.

In the Centigrade scale the melting point of ice is taken as the zero and the boiling point of water is called 100. In the Fahrenheit scale the distance between the melting point of ice and the boiling point of water is divided into 180 parts; the melting point of ice is not the zero, but is called 32°, and the zero probably marks what Fahrenheit considered the *lowest at-*

tainable temperature. If we suppose the same thermometer to have these two separate scales adjusted to it, or (still better) engraved side by side upon the tube, we easily see how to reduce from one scale to another. For if F and C be the two readings of a given temperature, it is obvious that

F − 32 bears the same ratio to (212 − 32), or 180
that C bears to 100

Hence $$\frac{F-32}{180} = \frac{C}{100}.$$

X

Boiling Temperature (t) of Water at Barometer Pressure (b).

b	*t.*	*b*	*t.*	*b*	*t.*	*b*	*t.*	*b*	*t.*
mm.	°	mm.	°	mm.	°	mm.	°	mm.	°
680	96·92	700	97·72	720	98·50	740	99·26	760	100·00
81	96·96	01	·76	21	·54	41	·30	61	·04
82	97·00	02	·80	22	·57	42	·33	62	·07
83	·05	03	·84	23	·61	43	·37	63	·11
84	·09	04	·88	24	·65	44	·41	64	·15
85	·13	05	·92	25	·69	45	·44	65	·18
86	·17	06	·96	26	·73	46	·48	66	·22
87	·21	07	97·99	27	·77	47	·52	67	·26
88	·25	08	98·03	28	·80	48	·56	68	·29
89	·29	09	·07	29	·84	49	·59	69	·33
690	·32	710	·11	730	·88	750	·63	770	·36
91	·36	11	·15	31	·92	51	·67	71	·40
92	·40	12	·19	32	·96	52	·71	72	·44
93	·44	13	·23	33	98·99	53	·74	73	·47
94	·48	14	·27	34	99·03	54	·78	74	·51
95	·52	15	·31	35	·07	55	·82	75	·55
96	·56	16	·34	36	·11	56	·85	76	·58
97	·60	17	·38	37	·14	57	·89	77	·62
98	·64	18	·42	38	·18	58	·93	78	·65
699	·68	19	·46	39	·22	59	·96	79	·69
700	97·72	720	98·50	740	99·26	760	100·00	780	100·73

XI

BAROMETRIC CORRECTIONS.

Correction for Temperature.

(Mercury—Brass scale correct at 0° C.)

Temp.	73	74	75	76	77	78	79
Degrees							
15 C.	0.178	0.181	0.183	0.186	0.188	0.191	0.193
16	0.190	0.193	0.196	0.198	0.201	0.203	0.206
17	0.202	0.205	0.208	0.210	0.213	0.216	0.218
18	0.214	0.217	0.220	0.223	0.226	0.229	0.231
19	0.226	0.229	0.232	0.235	0.238	0.241	0.244
20	0.238	0.241	0.244	0.247	0.251	0.254	0.257
21	0.250	0.253	0.256	0.260	0.263	0.267	0.270
22	0.261	0.265	0.269	0.272	0.276	0.279	0.283
23	0.273	0.277	0.281	0.284	0.288	0.292	0.296
24	0.285	0.289	0.293	0.297	0.301	0.305	0.309

Corrections are to be subtracted from observed readings—*viz.*, if reading at 19° is 76 centimetres, the "corrected" reading is 76—0.235 = 75.765 centimetres.

XII

Average Coefficients of Linear Expansion Between 0° and 100°C.

Aluminium	0.000023	Iron (cast)	0.000010
Brass	0.000018	Lead	0.000029
Copper	0.000017	Platinum	0.000009
Glass	0.000009	Silver	0.000019
Gold	0.000014	Steel	0.000011
Iron (soft)	0.000012	Tin	0.000022
Zinc	0.000029		

XIII

Specific Heats (*Approximate*).

(Gases at constant pressure).

Air	0.238
Alcohol at 17°C	0.580
Aluminium	0.218
Antimony	0.051
Bismuth	0.030
Brass	0.094
Carbon disulphide at 15°C	0.230
Chloroform at 15°C	0.233
Copper	0.093
Ether at 15°C	0.530
Glass	0.200
Glycerine, 0°—100°C	0.555
Hydrogen	3.409
Ice	0.504
Iron	0.112
Lead	0.031
Mercury at 15°C	0.033
Nickel	0.110
Oxygen	0.217
Platinum	0.032
Silver	0.056
Tin	0.056
Turpentine at 17°C	0.430
Water	1.000
Zinc	0.093

XIV

Melting Points and Boiling Points.

Substance.	Melting Point.	Boiling Point. 760 mm.
Alcohol (ethyl)		78°4C.
Alcohol (methyl)		66.5
Aluminium	606°C.	
Antimony	432	1450
Beeswax	62	
Benzine		90–110
Bismuth	260	1450
Bromine	—7	58
Carbon dioxide		—80
Carbon disulphide		46.2
Copper	1054	
Ether (ethyl)	—117.4	34.9
Glycerine	17	290.
Hydrogen		—243.
Ice	0	
Iodine	114	
Iron	1600	
Lead	325	1500
Mercury	—39	357
Nickel	1450	
Oxygen		—184
Paraffine	52–56	
Phosphorus	44	283
Platinum	1775	
Silver	954	
Sulphur	115	
Tin	230	
Turpentine		159.3
Water		100
Zinc	412	

XV

Latent Heats.

Substance.	Fusion.	Vaporization.
Alcohol (ethyl)		202.4
Alcohol (methyl)		264.0
Beeswax	97.2	
Bismuth	12.6	
Bromine		45.6
Carbon disulphide		86.7
Ether (ethyl)		90.4
Ice	79.2	
Iodine	11.7	
Lead	5.8	
Mercury	2.8	62.0
Nickel	4.6	
Phosphorus	4.7	
Platinum	27.2	
Silver	24.7	
Sulphur	9.4	
Tin	14.6	
Turpentine		74.0
Water		535.9
Zinc	28.1	

XVI.

Indices of Refraction (for D line).

Air	1.0003	Glass, flint	1.63
Alcohol	1.363	Glass, crown	1.52
Carbon disulphide	1.624	Ice	1.3
Diamond	2.47	Water	1.334

XVII

Electromotive Force of Cells.

CELL.	E.	CELL.	E.
Bunsen	1.68	Grenet (Chromic Acid),	1.8
Clark, at 15°	1.434	Grove	1.84
Daniell, copper sulphate		LeClanché	1.4
and 9% sulphuric acid,	1.078	Planté (Storage) . . .	2.0
Daniell, Gravity . . .	(about) 1	Water Element . . .	1.0

XVIII

Specific Electrical Resistances.

Copper	1.6×10^{-6}	Nickel	12.3×10^{-6}
German Silver	20.8×10^{-6}	Platinum	8.9×10^{-6}
Iron	9.6×10^{-6}	Silver	1.5×10^{-6}
Mercury	94.3×10^{-6}		

XIX

Electro-Chemical Equivalents.

Weight in grammes deposited by a current of 1 ampere per second.

Copper (cupric) = 0.000328
Silver = 0.00118
Iron (ferric) = 0.000103
Hydrogen = 0.00001038
Oxygen = 0.0000828

Volume in cubic centimetres liberated at 0 deg. C. and 760 mm

Hydrogen = 0.1156
Oxygen = 0.0578

XX

Natural Sines and Tangents.

DEG.	TANG.	SINE.	DEG.	TANGENT.	SINE.	DEG.	TANGENT.	SINE.
1	.017	.017	31	.601	.515	61	1.80	.875
2	.035	.035	32	.625	.530	62	1.88	.883
3	.052	.052	33	.649	.545	63	1.96	.891
4	.070	.070	34	.675	.559	64	2.05	.899
5	.087	.087	35	700	.574	65	2.14	.906
6	.105	.105	36	727	.588	66	2.25	.914
7	.123	.122	37	.754	.602	67	2.36	.921
8	141	.139	38	.781	.616	68	2.48	.927
9	.158	.156	39	.810	.629	69	2.61	.934
10	.176	.174	40	.839	.643	70	2.75	.940
11	.194	.191	41	.869	.656	71	2.90	.946
12	.213	.208	42	.900	.669	72	3.08	.951
13	.231	.225	43	.933	.682	73	3.27	.956
14	.249	.242	44	.966	.695	74	3.49	.961
15	.268	.259	45	1.000	.707	75	3.73	.966
16	.287	.276	46	1.036	.719	76	4.01	.970
17	.306	.292	47	1.070	.731	77	4.33	.974
18	.325	.309	48	1.110	.743	78	4.70	.978
19	.344	.326	49	1.150	.755	79	5.14	.982
20	.364	.342	50	1.190	.766	80	5.67	.985
21	.384	.358	51	1.230	.777	81	6.31	.988
22	.404	.374	52	1.280	.788	82	7.12	.990
23	.424	.390	53	1.330	.799	83	8.14	.993
24	.445	.407	54	1.380	.809	84	9.51	.995
25	.466	.423	55	1.430	.819	85	11.43	.996
26	.488	.438	56	1.480	.829	86	14.30	.998
27	.510	.454	57	1.540	.839	87	19.08	.999
28	.532	.469	58	1.600	.848	88	28.64	.999
29	.554	.485	59	1.660	.857	89	57.29	1.000
30	.577	.500	60	1.730	866	90	Infinite.	1.000

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